GAU's analog calculator: multiply, divide, square, and extract roots

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Introduction

The slide rule has always been the distinguishing symbol of every true engineer, much like the drum is for the shaman. Over time, desktop computers, pocket calculators, and personal computers have made their way to our desks, but the slide rule's initiatory allure has remained unsurpassed.

I recently read an article describing an analog calculator in an old magazine, guarded by an acquaintance more jealously than a Leonardo da Vinci manuscript. The magazine was the July 1968 issue of Sperimentare, and the article was titled "Archimedes, Electronic Calculator."

I did some research online, but aside from numerous articles on simulated analog computing with electronic components, I couldn't find anything this simple. I think it's worth reproducing the circuit because it's extremely interesting. In the documentation, I've included a photo of the original schematic, taken with my cell phone while the stern librarian was distracted.

I repeat that the idea is absolutely not mine and I extend my respectful acknowledgement to the author of the article, whose initials "GAU" are unfortunately only reported, for the ingenious solutions adopted in a circuit of rare elegance in its simplicity.

What does the GAU analog calculator do?

With a 9 Volt battery, three potentiometers, a microammeter and little else, it performs additions, subtractions, multiplications, divisions, squares and square roots of numbers.

ranging from zero to ten with a precision of one decimal place. It has three graduated dials for setting the numbers on which to calculate and reading the result.

The cornerstone of the circuit is the independence of the calculation results from the battery voltage: you can go from 1.5 to 18 V volts without any problems.

In designing it, I eliminated the switch that allows additions and subtractions, operations not present in slide rules, because these operations require an additional battery with absolutely identical voltage to the main one: the results would not be constant over time. Instead, I added the ability to increase the precision of division operations if the quotient is greater than 10: without this feature, the calculator would have used only a tenth of the useful travel of a potentiometer. It's a small "retrofit" that only marginally improves the original design.

Operations on all numbers

The GAU calculator works on numbers between 0 and 10 with two significant digits.

What if I have to multiply 2,456 by 435,469?

The trick is to convert the factors to be multiplied into numbers in exponential notation and calculate the normalized fractional part (or mantissa) and the exponent (or characteristic) of the result separately.

Don't say you don't know what you're talking about, otherwise how can you calculate the time constant of an NE555 with a 6.8 nanofarad capacitor and a 120K resistor? The mantissas are 6.8 and 1.2 while the exponents are (– 9) and (+5).

Before the advent of pocket calculators, every designer in the world worked with slide rules this way, think about it, folks!

multiplication 2,786 x 432,469 = ????

2.786 = 2,8 * 10^3
432.469 = 4,3 * 10^5

2,8 * 4.3 * 10^(3 + 5) = 12 * 10^8 = 1,200,000,000 (exact value 1,204,858,634)

the division 75.826 / 471 = ???

75.826 = 76 * 10^ 3
471= 4,7 * 10^ 2

76 / 8.7 * 10^ (3-2) = 16 * 10^1 = 160 (exact value 160.9893843)
The square of 6.521^2 = ????

The square is the multiplication of a number by itself.

6.521 = 6.5 x 10^3

6.5 x 6.5 x 10^(3 x 2) = 41 * 10^6 = 42,000,000 (exact value 42,523,441)

The square root radq (264.247) =???

the square root <a> of a number <b> is the number <a> that multiplied by itself gives the number <b>

264,247 = 26 x 10^4 (the number must be written with the EVEN exponent!)

square root (26 x 10^4= square root(26) x 10^(4/2) = 5.1 x 10^2 = 510 (exact value of 514.0496)

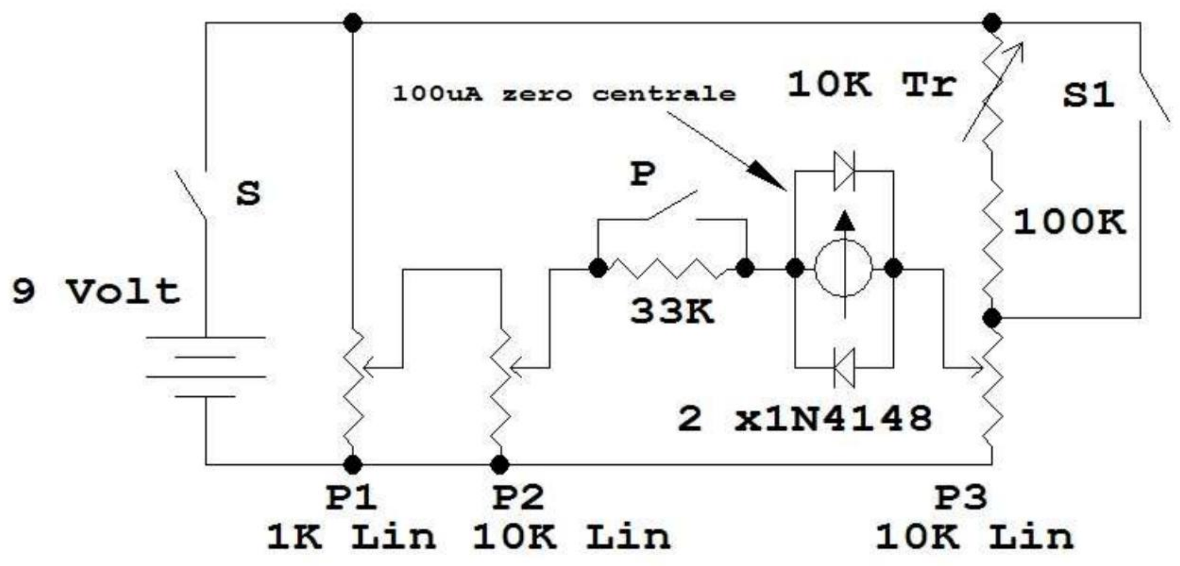
The accuracy of the results

The theoretical accuracy of calculations using numbers rounded to two significant figures averages 1.2% (in 10% of cases, it drops to 2.5%). This applies to values that are beyond 10% of the reading scale (1.0 for a full scale of 10 and 10 for a full scale of 100). Using the first 10% of the scales significantly reduces accuracy: it is precisely for similar reasons that textbooks recommend always taking measurements near the full scale of the instruments.

We take into account the nonlinearity of the potentiometers due to loads, scale misalignment, travel tolerance, pin backlash, reading approximations, and more. To be on the safe side, we reduce the result fourfold (as engineers designing bridges often do), but we still get a precision close to 5%. These seem like interesting values, considering that most components have a nominal tolerance of 10% (for example, E12 series resistors and capacitors): in electronics calculations, with rare exceptions, a precision greater than 5% is useless.

The circuit

The heart of the device is made up of three potentiometers and a microammeter. The other components are simple accessories.



Compared to the original diagram, I added the 10K trimmer, the 100K resistor and the switch S1 which, in the "open" position, multiplies the reading scale of P1 by 10.
Instead, I eliminated the auxiliary battery for additions and subtractions with the related switch.

How it works

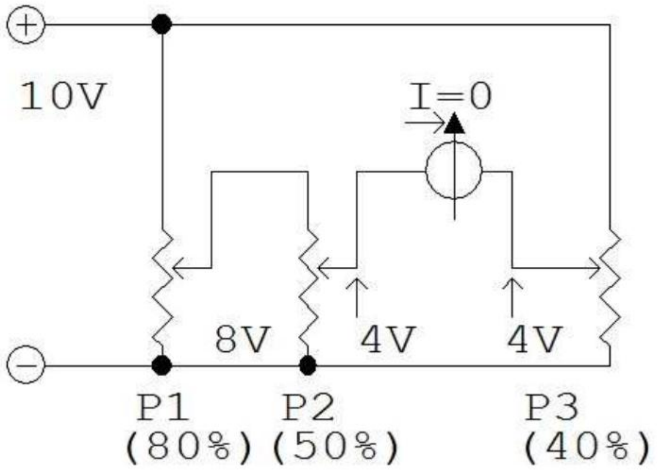
We want to multiply 8 by 5.

For the explanation we use a 10 Volt battery, so the calculations are more intuitive.

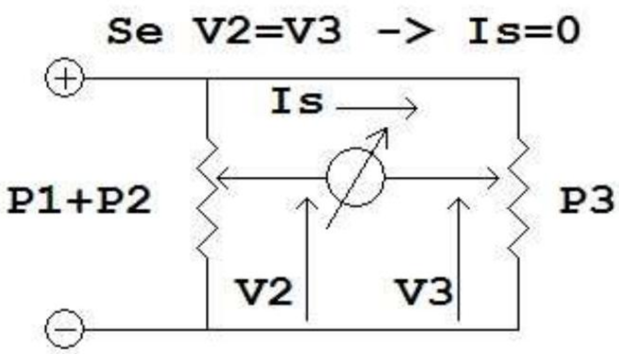
The battery powers the linear variation potentiometer P1, graduated in 10 divisions and positioned at 80% of the travel: at its output we have a voltage of 10 x 0.8 = 8 Volts which powers the potentiometer P2.

The linear potentiometer P2, graduated on 10 divisions, is positioned at 50% of the travel and at its output we have a voltage of 8 x 0.5 = 4 Volts.

If we ignore the zeros and decimal points, this voltage is the product of the positions of potentiometers P1 and P2: we need to read this voltage somehow.



Potentiometer P3 is connected to the battery. Its output voltage is proportional to the cursor position. If its scale is divided into 100 graduations, then setting it to <40> will result in a 4 Volt output.



To verify that the two voltages are actually equal, check that no current flows in the central zero microammeter; with the instrument zeroed, we have this situation:

	scale position partition ratio 0.8 0.5 0.4	**voltage on the way out**
P1 0- 10	**8**	**10 x 0.8 = 8 V**
P2 0- 10	**5**	**8 x 0.5 = 4 V**
P3 0-100	**40**	**10 x 0.4 = 4 V**

So reading the indicators on the potentiometer scales we have

$$8 \times 5 = 40$$

This result is always valid, regardless of the battery voltage: simply set the factors to be multiplied on P1 and P2, zero the instrument by adjusting P3 and read the result on P3. The two left potentiometers P1 and P2 "perform the multiplication" and, once the instrument is zeroed, the right potentiometer P3 "reads the result": to say that the GAU calculator is brilliant is an understatement.

My retrofit for the division

If the quotient is less than 10, all is well, however, in half the cases it is greater than 10. When this happens, to still have a quotient less than 10, you must position the dividend potentiometer P3 in the first 10% of the scale. As mentioned before, this is the low-precision area, but with a little trick you can "multiply by 10" the scale of the quotient potentiometer P1.

For example for 74.212 / 552 = ???

74.212 = 74 *10^3

552 = 5,5 * 10^2 74 /

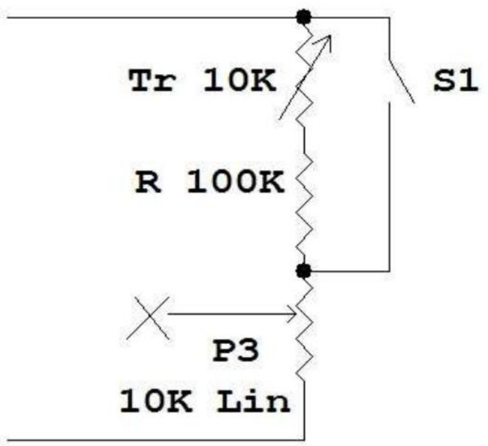
5,5 * 10^(3-2) = 13 * 10^1 = 130

the "normalized" quotient is <13> higher than full scale

To reset the instrument, you would need to set the dividend as <7.4> (very close to the beginning of the scale) instead of 74: the quotient read becomes <1.3>.

To avoid this, a resistor is inserted in series with potentiometer P3 to reduce the voltage across the potentiometer to 90% of the battery voltage. This is done using the 10K multi-turn trimmer Tr and the 100K resistor R.
The theoretical value of (Tr+R) is equal to 9 * P1, but the required value is higher and will need to be adjusted during calibration.

S1 "aperto" P1=DIVx10



The scale of P3 is "expanded" and the old value <7.4> is now at the <74> position; consequently the scale of P1 (the quotient) must also be multiplied by 10. In summary, set the dividend (potentiometer P3) to <74>, the divisor (potentiometer P2) to <5.5> and with switch S1 in the "open" position DIV x10, read the quotient <13> on potentiometer P1 by multiplying its scale by 10.

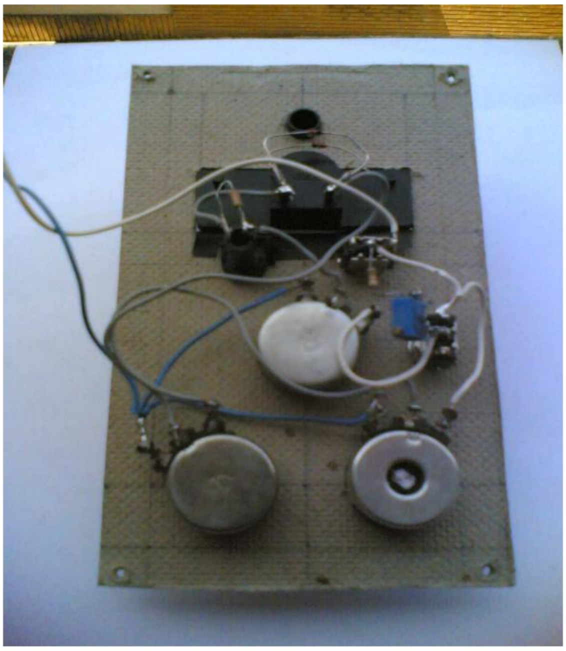
Some practical considerations

- The battery voltage, whatever its value and even if it varies over time, does not affect the results: this is the key to the GAU calculator - the potentiometers must be wire-wound. The nominal linearity of these components is +/- 0.25% (a bit overly optimistic!)

- The value of P2 must be at least 10 times that of P1: in this way the loss of linearity of P1 due to the load of P2 is 2.5% - the resistance value of the
potentiometers does not affect the results - with P1 = 1K, P2 = 10K, P3 = 10K
and a 9 Volt battery the total absorption is about 11 mA, less than an LED - since we are making a "zero measurement" the
instrument does not have to be high quality or even linear, it just needs to be sufficiently sensitive
- The instrument is protected by the 33K resistor in series which limits
the current to about 300 uA in the worst case, the two diodes protect it further by limiting the voltage to +/- 0.7 volts - The P button short-circuits the resistor and allows you to have a precise "zero" using the maximum
sensitivity of the instrument, nothing new: all decade bridges were like this.

The assembly

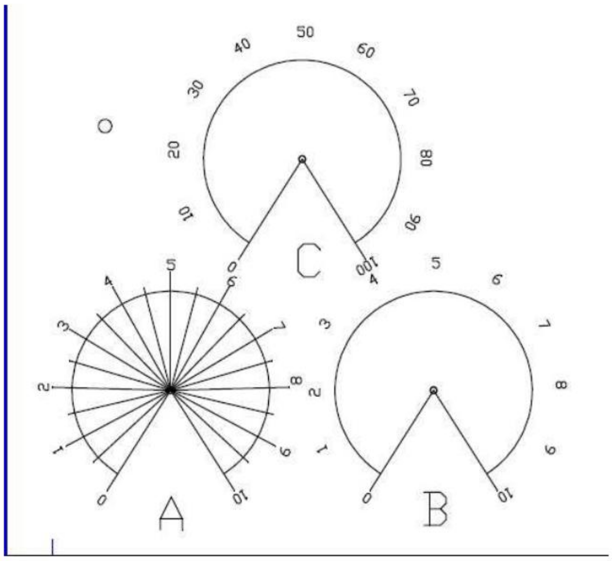
By housing the calculator in a 15 x 10 x 5 cm plastic box, simply connect the terminals of the potentiometers, the meter, and the switches mounted on the front panel with an insulated wire. In the prototype, I used a sheet of pressed cardboard to later use as a template for the final metal panel.



On the right in blue you can see the multi-turn trimmer soldered onto the S switch. The wires coming out of the circuit go to the battery.

The reading dials and their calibration

Potentiometers P1 and P2 have a graduated scale from 0 to 10; potentiometer P3 has a graduated scale from 0 to 100.



To draw the dials you can use a drawing software: I used ProgeCAD2009Smart which is released free for strictly non-commercial use.
I also used it to sketch the front panel.

The dial of P1 must be drawn with regularly spaced subdivisions, while that of P2 and P3 will only show the arc with the start and end marks: the subdivisions will be made by hand during calibration.

Calibration of the reading dials

The original article recommends manually calibrating potentiometer scales because this corrects many systematic errors. Calibration for multiplication is sufficient.

The procedure must be carried out with the S1 switch in the "closed" position DIV x1.

a) calibration of the potentiometer scale P1

the scale remains the “geometric” one with the notches from 0 to 10 and the subdivisions equally spaced in steps of 0.1.
Check in advance the actual rotation amplitude of P1 to allocate the divisions over the entire useful arc.

b) calibration of the potentiometer scale P3

1) set potentiometer P2 to full scale on division <10> 2) set potentiometer P1 to division <0.5> 3) adjust potentiometer P3 until the instrument is zeroed 4) adjust potentiometer P3 with the P key pressed until the instrument is zeroed 5) mark the value <5> on the scale of potentiometer P3 6) repeat operations 2) to 4) for all the values of P1 in steps of 0.5 from <1.0> to <10> marking the corresponding points on P3 from <10> to <100> in steps of <5>

7) now the P3 scale is calibrated on 20 points <0-5-10-15...95-100>.

c) calibration of the potentiometer scale P2

1) set potentiometer P1 to full scale on division <10> 2) set potentiometer P3 to the previously marked division <5> 3) adjust potentiometer P2 until the instrument is zeroed 4) adjust potentiometer P2 with the P key pressed until the instrument is zeroed 5) mark the value <0.5> on the scale of potentiometer P2 6) repeat operations 2) to 4) for all the values of P3 in steps of 5 from <10> to <100> marking the corresponding points on P2 from <1> to <10> in steps of <0.5>

7) now the P2 scale is calibrated on 20 points <0-0.5-1.0-1.5...9.5-10.0>.

Further subdivisions in steps of 0.1 for P2 and in steps of 1 for P3 can be done by hand without losing too much linearity.

Retrofit calibration

1) set switch S1 to the "open" position DIVx10 2) using the already calibrated scales, set the value <3> on P1, the value <3> on P2, the value <90> on P3 3) connect a digital voltmeter to the central terminal of P2 4) press the P button, read and note the exact value of Vp2 (e.g. 0.760 Volts or whatever) 5) connect the digital voltmeter to the central terminal of P3

6) press the P button and read the voltage Vp3
7) with the P button pressed adjust the Tr trimmer until Vp3 is exactly equal to Vp2 (e.g. 0.760 Volt +/- 0.01) 8)
place the S1 switch in the "closed" position DIVx1 9) now
the DIVx10 scale of P1 is calibrated

The procedure seems abstruse, but it's only a "zero" measurement. Starting from a known equilibrium position (30 x 3 = 90) on already calibrated scales, the corresponding voltages are equalized. It's not possible to calculate this "zero" with the calculator's microammeter because it's not sensitive enough:

The calculator is ready: how do you use it?

Data preparation

As mentioned, it is necessary to bring all the data into exponential notation in the form

potentiometer P1 and P2 “Unit, Decimal” x 10 ^ “exponent” e.g.: 4.3x 10^5
potentiometer P3 “Tens-units” X 10 ^ “exponent” e.g. 33 x 10^2

It doesn't take long to get the hang of it.

Multiplication

make sure that switch S1 is "closed", position DIV x1

Set the first factor to P1 (Unit, Decimal)
Set the second factor to P2 (Unit, Decimal)
Adjust P3 to zero the instrument
Press P and P3 to fine-reset the instrument
Read the result on P3 (Tens-Ones)
Add the exponents

Division

make sure that switch S1 is "closed", position DIV x1

Set the dividend to P3 (Tens-Ones)
Set the divisor to P2 (Units, Decimal)
Adjust P1 to zero the instrument If
the instrument does not zero: move switch S1 to the "open" position DIV x10
Adjust P1 to zero the instrument
Press P and adjust P1 to finely zero the instrument Read the
result on P1 a) if S1 is
on DIV x1 read (Units, Decimal) b) if S1 is on DIV x10
read (Tens.Units) make sure to always return the
switch to the "closed" position DIV x1
Subtract the exponents

Squaring

make sure that switch S1 is "closed", position DIV x1

Set the number to square to P1 (Units, Decimal)

Set the number to square to P2 (Units, Decimal)
Make sure that the positions of P1 and P2 are as equal as possible.
Adjust P3 to zero the instrument
Press P and adjust P3 to fine-zero the instrument
Read the result on P3 (Tens-Units)
Multiply the exponent by two

Square Root Extraction

Warning: The exponent must be an EVEN number
234,534 should be written as 23 x 10^4 and NOT as 2.3 x 10^5

make sure that switch S1 is "closed", position DIV x1

Set the number to square root to P3 (Tens-Ones)
Set P1 and P2 to the same value to zero the instrument (Unit, Decimal)
Make sure that the positions of P1 and P2 are as equal as possible to each other.
Press P and finely adjust P1 and P2 again to zero the instrument.
Make sure that the positions of P1 and P2 are as equal as possible to each other.
Read the result on P1 (Unit, Decimal)
Divide the exponent by two

The first "accounts" with the GAU calculator

The prototype scales were designed with subdivisions only every 0.5 points for P1 and P2 and every 5 points for P3, corresponding to the 20 calibration points: I estimated the rest by eye.

I spent an evening doing a lot of calculations involving multiplication, division, squaring, and root calculations. The results were accurate to within 4% to 6%. I then had my assistants perform the calculations, and the accuracy increased to 2% to 3%, confirming the importance of proper dial positioning and accurate reading estimation.

The results are consistent with the assumption that an accuracy of 5% was expected. By drawing all 100 subdivisions on the scales, we can certainly improve to below 2%. I'd say I'm more than satisfied!

How it works: a few more formulas

1) The voltage at the output of a potentiometer Ra+Rb loaded by a resistor Rc is

$$Vout = \frac{Rb \times Rc}{W \times Rc + Rb \times Rc + W \times Rb} \times Vbatt$$

If Rc is sufficiently greater than Ra and Rb we will have the well-known formula

$$Vout = \frac{Rb}{Ra + Rb} \times Vbatt$$

Which we can write as

Vout = K x Vbatt, where K is the partition ratio ranging from 0 to 1

2) At the output of the first section of the potentiometer P1 we have a voltage equal to

V1 = Vbatt x K1

which powers the potentiometer P2

3) At the output of potentiometer P2 there is a voltage equal to

V2= V1 x K2 = Vbatt x K1 x K2

which goes to one end of the microammeter

4) At the output of potentiometer P3 the voltage will be

V3= Vbatt x K3

which goes to the other end of the microammeter

5) The circuit is an extreme simplification of the "potentiometric" measurement method: the voltages of two nodes are equalized, verifying that no current flows between them. P1 and P2 generate the unknown voltage while P3 generates the known voltage.

In this "zero current" condition V3= V2

and therefore based on points 3) and 4)

Vbatt x K1 x K2 = V2 = V3 = Vbatt x K3

6) We divide everything by Vbatt (this can be done because Vbatt is non-zero), making it disappear from the formula: this is why the battery voltage is irrelevant. Finally, we get:

K1 x K2 = K3

"And what does this mean to me?", Camilleri would say, this means:

Since the potentiometers are linear and the output voltages correspond to the position of the sliders then

Position (P1) x Position (P2) = Position (P3)
The GAU calculator performed the multiplication.

Furthermore, if K1 x K2 = K3 then K1= K3 / K2, so just position the indexes of potentiometers P3 and P2, zero the bridge with the instrument and read the result on the index of potentiometer P1 to have the result of the division.

Position (P1) = Position (P3) / Position (P2)

The GAU calculator performed the division.

"Perhaps you didn't think I was a logician," said the good demon.

Possible developments

– connect "follower" op-amps to the output of all potentiometers – use a double potentiometer, with the op-amps, for the square and the root – use multi-turn potentiometers with their expensive counter knobs
– replace the instrument with an operational amplifier with two LEDs on the output
– draw scales that directly connect **Xc = 1 / (2 ÿ xfx C) and Xl = 2 x ÿ xfx L**
– etc, etc

The list of components

nr. 1 1K 5W wire-wound linear potentiometer nr. 2
10K 5W wire-wound linear potentiometers nr. 1 100uA
center-zero microammeter nr. 2 1N4148 diodes nr. 1 33K ¼
W resistor nr. 1 100K ¼
W resistor nr. 1 10K multi-turn
trimmer nr. 1 Pushbutton nr. 2
Switches nr. 1 9 Volt battery nr.
3 Knobs with
index nr. 1 Container

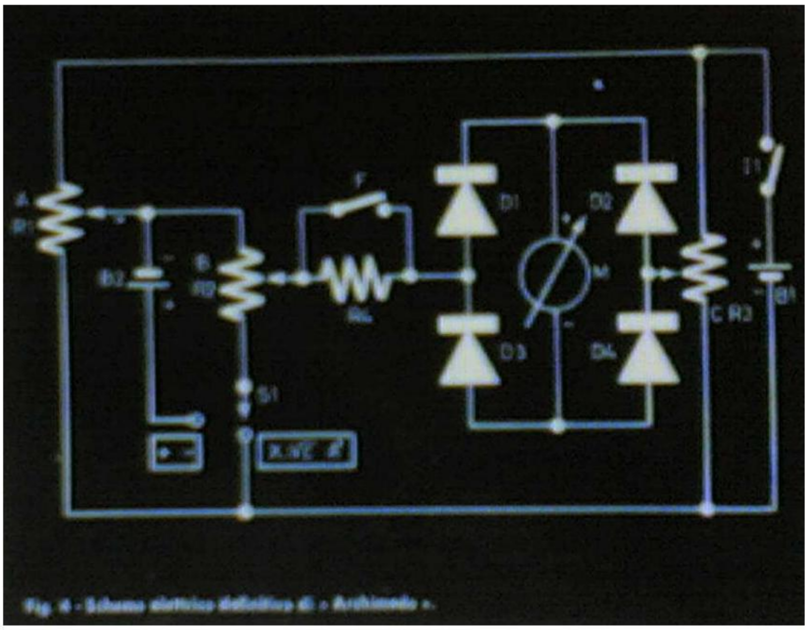
The estimated costs

The three potentiometers 18 euros
The 10 euro microammeter
button, switches, knobs 2 euros
diodes, resistors, trimmers 2 euros
The box 35 euros **3 euro**
Total

Availability of components

I would say there are no problems.

A photo of the original 1968 schematic



Note the switch at the bottom left that connects the auxiliary battery partitioned from P2 in series with P1: it is used for addition and subtraction. The alternative use of a microammeter without a central zero and a germanium diode bridge was suggested: it was 1968!

The documentation

Article from GAU Sperimentare, July 1968: "Archimedes, electronic calculator"
progeCad2009 smart http://www.progesoft.com/en/smart-2009
analog computing http://www.electroportal.net/g.schgor/wiki/articolo11
Analog Calculators http://www.tecnoteca.it/museo/05
Analog computers http://en.wikipedia.org/wiki/Analog_computer

Conclusion

You might ask: was all this fuss really necessary, given that a calculator that calculates with eight digits costs a couple of euros? True, but the same can be said about the sextant, given that there's GPS and CW now that cell phones are available even in Ngoro Ngoro. I may only use GAU's analog calculator once in a while, as I do with my astral compass and my bubble sextant, but I had a lot of fun calculating a square root with three potentiometers.