Analog Computer Prototyping for the Future

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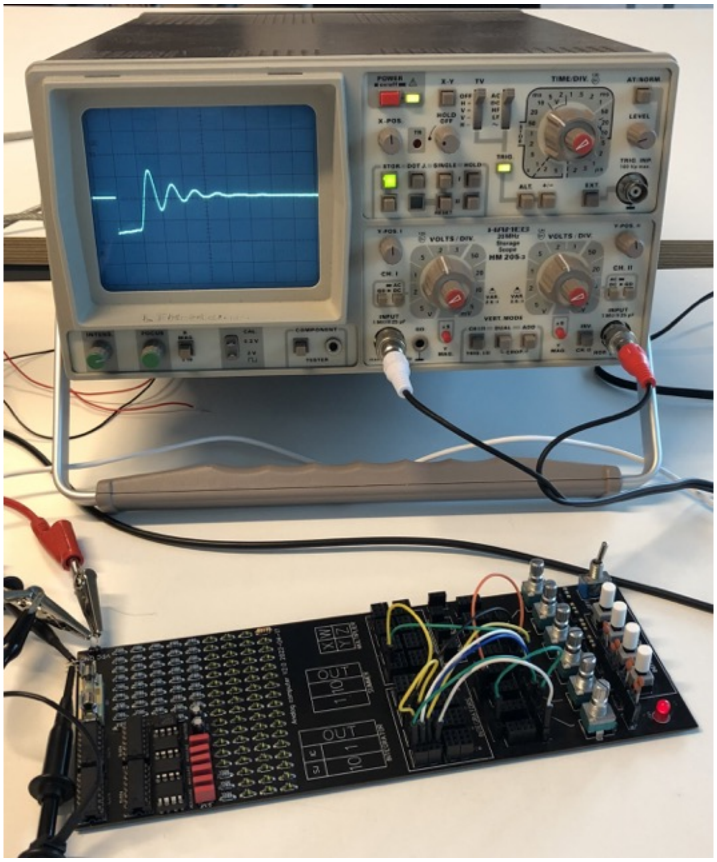

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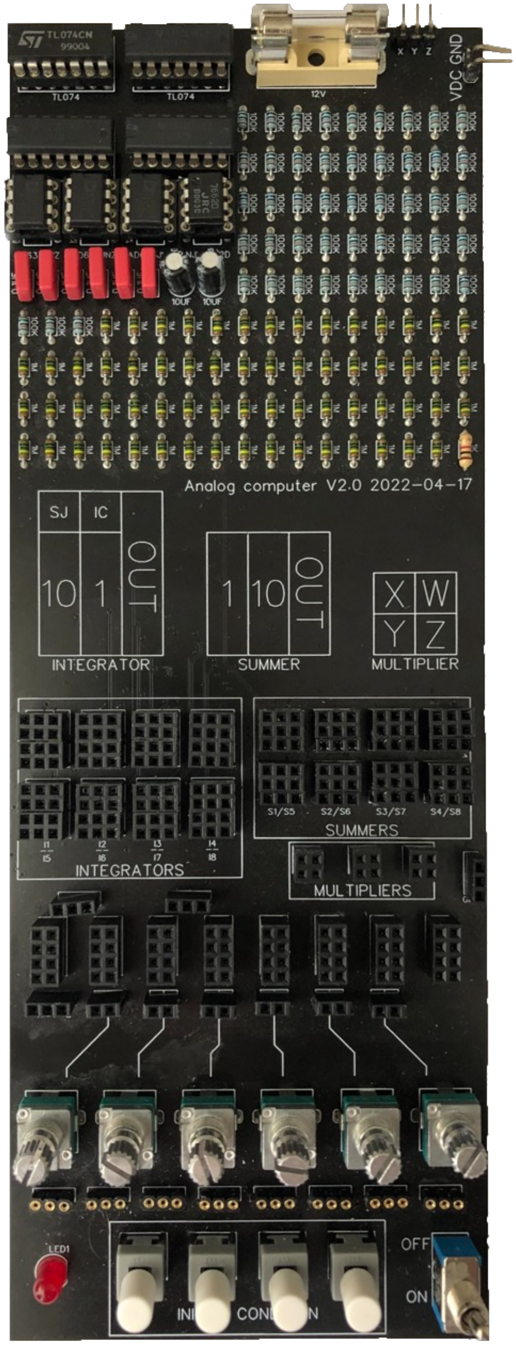


The analog computer created during this project.

CONTENTS

***Abstract*—This research paper focuses on analog computers and creating a modular low-cost analog computer system in a single board computer form factor. The single-board analog computer will have the capacity to solve second-order differential equations. The capabilities and possibilities of the single board Analog computer will be explored as well as analog computing in general. The paper follows design science research methodology (DSRM) with the goal of creating and evaluating a working artifact. The artifacts' functionality is evaluated based on a demonstration of its ability to solve Mathieu's differential equation as well as simulate a spring-mass dampening system. This paper proves that it is possible to create a low-cost analog computer in a modern form factor. The artifact is also placed in a larger contextual setting based on the empirical material provided where its value of it in a digital society is presented. For the world to continue its progression in computational power, but still, limit the already high energy usage, a drastic change is needed. This paper suggests adapting to analog/hybrid technology. To further the progression of analog/hybrid technology it must be made accessible to a wider group of people compared to today. The artifact in this paper offers a solution to this.**

***Index Terms*—Analog computing, Hybrid systems, Differential equation, Single board computer, DSRM, Sustainable IT**

I. INTRODUCTION

Digital computers are the norm today regarding computational power. Since the solid-state transistor became widely available, the digital computer has been dominating computation. Today's high energy consumption and potential stagnating progression of digital computers have led to a new turning point in the progression of computational speed and sustainable IT. In order to meet the global demand for greater computing power, a drastic change has to be introduced to the world of computation. This is a must if we wish to continue our present computational progression and decrease our ecological footprint. One of the possible solutions to this problem is to change the fundamental view of computation [1].

By switching to solving problems using analog computing a positive impact could be achieved in today's ecological and computational scene compared to using strictly digital systems. By creating hybrid systems that can utilize an analog coprocessor, that is optimal for solving differential equations, low-energy analog computing can be utilized to a larger extent compared to today. In order to achieve this, a new approach to introducing the world to analog computing is needed [2] [3] [1].

II. BACKGROUND

The three major computational paradigms are analog, digital and quantum [4]. The Analog computer has seen a steep reduction in prominence during the last 40-50 years due to the rapid improvement in digital computing. This has led to the fact that very little recent research has been conducted on the analog computer and its potential role in today's society [3].

The popularisation of digital computers in today's society stems back to the early days of computing when both analog and digital computers were common (the following decade after the release of Vannevar Bush's analog computer). The early digital computers relied on the user writing code to solve a problem algorithmically, as the case is still to this day. The early problem was that the digital machines often were slow and expensive, thus hindering the user to solve problems. The digital machines were and still are based on an algorithmic sequence thus not being able to run many calculations simultaneously natively.

Analog computers have a long history dating back to around 1000 years ago. The most famous example of this age is the Antikythera mechanism dated to around 100 B.C [5].

One of the more recent famous analog computers is the Vannevar Bush's differential analyzer released in 1931. The analog computer Bush created was initially meant to model power networks, but its value as a general-purpose analog computer was quickly realized. The computer was a large mechanical construction made of an assortment of gears and shafts driven by electric motors. The disc wheel based integrators could be connected to a number of rotating shafts. The computer could solve sixth-order differential equations, although this was complex to set up. The machine was used in solving problems in physics, seismology, and ballistics [6] [7]. These early examples are all examples of mechanical analog computers.

After the release of Bush's analog computer, the electric computer was on the horizon. The electric analog computer was widely developed by many manufacturers during this time including, Electronic Associates Inc., Applied Dynamics, RCA, and Telefunken among others. The electric analog computer was first used as an aid in missile and airplane design as well as running aerodynamics calculations. This meant that the space and aviation industries were the largest customers during this time. The customer base later extended to nuclear-reactor control [8] [9].

Analog computers are not high-accuracy devices. Therefore their usage was not centered around tasks that required high levels of accuracy. The analog computer was used often in engineering and applied sciences as an analog for the problem being studied. One of the most common and least complex calculations is the linear ordinary differential equation (which is described in the terminology section of this paper) [3].

Physics calculations are very well suited for analog computation. In the early days of space flight, analog computers were used to calculate orbit trajectory and other orbit-related equations such as satellite positioning [10]. Calculating particle trajectories was also a well-established field within physics where analog computers were widely used [11] [3]. See fig 1 for an visual example of an commercial electro analog computer.

Before the invention of the modern MOS (Metal Oxide Semiconductor) the electric analog computer was based on vacuum tube technology. The vacuum tubes acted as transistors and were later replaced by modern solid-state transistors. The programming of the analog computer was (and still is to some extent) done by interconnecting different computational modules through a patch panel. This way of programming

Fig. 1. GTE Analog Computer EA22 [12]

is very complex and required a skilled programmer to be successful. This is also one of the reasons for their later demise. Another contributing factor to the demise of the analog computer was the introduction of the digital computer. The more straightforward approach to programming, algorithmic computational structure, the ability to easily store information, high precision, and ability to handle any problem given the increase in computational time was a clear advantage over the analog computer. The advancements of MOS and chip technology made digital transistor-based chips faster and cheaper thus quickly rendering analog computers obsolete [8] [5].

This paper will not focus on pre-electric analog computers like the Antikythera mechanism and the Vannevar Bush's differential analyzer. The focus will lie on electric analog computers, for example, the computer created by the research team at Anabrid [4]. Anabrid has created an open-source analog computer that aims to enable researchers and students to explore the area of analog computing [13]. The device can calculate and simulate several large computational challenges like market economics, the spread of disease, and more [13].

Digital computers are approaching the end of continuous improvement and Moore's law (where the computer chips can not be made faster by physical means) will take full effect. To keep progress going, something has to be done in order to further the improvements in computational power, since quantum computing is still far off in the future as a viable option. This could be done by creating a hybrid computer that takes all the good aspects to form digital and analog computers and combines them into a hybrid computer like Steiglitz [14] proposes.

One of the most challenging parts of analog computing in today's society is the user experience, availability, and ease of use. In order to have analog computing viable in today's society, a better way is needed to interact with the computer.

A. Relevancy

The global energy consumption by data centers, servers, and other digital hardware is staggering. In 2017, Germany alone used 13.2 billion kWh of electricity to power data centers and servers. This was an increase from 12.6 billion Kwh in 2016. The global consumption is estimated to be 350 billion Kwh in 2017. Given the fact that there has been a steady increase in energy consumption between 2010 and 2017, there is evidence that points toward this trend continuing [15]. Most of the high energy usage is due to today's power-hungry CPUs. A warehouse-scale data center's total power usage is approximately 33-60% only due to the modern CPU [16].

The digital computer running boolean algebra is a versatile and well-developed system. One of its problems is the aforementioned high energy consumption of modern processors. A hybrid system that utilizes an analog co-processor has been proven to run faster and use less energy than a common digital processor as proven by Köppel et al [2]. The digital computer is also approaching the theorized Moore's law where the computer chips can not be made faster by physical means [2].

Analog computing has been lost to history due to the digital computer being more versatile and becoming easier to use. The three major computational paradigms are digital, analog, and quantum computing. Quantum computing is still too far away to be implemented in today's society. Therefore looking to history and investigating analog computation could be a way forward and continue the computational progression and create more energy-efficient systems.

III. PURPOSE AND GOAL

An analog computer will be created that has the capability to run different first and second order differential equations. The actual component-based computation that is based on the mathematical expressions for the differential equations will be converted into circuits. These circuits will then be tested and used as a tool to verify the functionality of the computer.

The purpose of this paper is to create an analog single-board computer to make analog computing accessible for academia and educational institutions. A low-cost, easy-to-use, analog computer can be used as an educational tool to further the innovation within analog technology. Given the fact that most analog computers are either expensive or non-existent on the open market, a low-cost modular analog computer could have a similar impact as the Arduino single-board computer platform had on academia. The created artifact will also be placed in a larger contextual setting where its role in a larger socio-material context will be investigated. The uses for the artifact and general analog computing in a digital society will also be investigated based on the empirical material provided in this paper. A hybrid setup using an Arduino and the analog computer as a co-processor will also be demonstrated. Therefore this project has the potential to be an innovation for change in a digital society for a more computation demanding and greener future.

IV. RESEARCH QUESTIONS

Based on the information presented in this paper thus far, the research questions for this project are as follows:

"RQ1 - What is the feasibility of creating a low cost modular single board analog computer that can solve second order differential equations?"

"RQ2 - What are the capabilities and usages of a low cost modular single board analog computer?"

V. RESEARCH METHODOLOGY

The methodology used in this paper is heavily influenced by design science research (DSR) and its methodology as described by Peffers et al. [17]. The focus of the research conducted is to create an artifact that has the potential to further the development of analog computers, therefore being actually useful in its domain. The methodology does not focus on rigid design processes and formal theory.

The design science research methodology (DSRM) presented by Peffers et al. [17] is influenced by the DSR forerunners March and Smith [18], Nunamaker [19], and Walls [20]. These authors were focusing on building actual artifacts that would later be used in case studies to examine their impact on a specific institution.

A DSRM research project is well suited to start with a loosely specified problem such as a stakeholder request or further development on an existing product. Important to note is that this method of conducting research is more of a suggestion on "how to do it" rather than stating that it is the best way to go about that particular research according to Peffers et al. [21].

The role of theory for DSRM is to create a technical base for the artifact being created to rest upon. Very little contextual theory is included since it is assumed that the artifact contains the context needed in order to justify its creation. This means that more focus can be allocated to the technical base for the artifact and limit contextual inclusions to a DSR paper. Although the artifact created holds context by its very creation, some context will be provided given the lack of prevalence of analog computing in today's society.

To evaluate the artifact, DRSM uses demonstration-based evaluation in order to prove the functionality of the artifact. If the artifact can be demonstrated using a predetermined functionality test, the functionality of the artifact can be proven instantly. If it can be demonstrated over a range of contexts, it can be evaluated [17].

In order to create the artifact, that is the low-cost analog computer, the different computational elements that make up an analog computer need to be investigated. This includes the analog computer programming schematics that is prevalent in a lot of textbooks from the time of the analog computers' height of popularity. These computational elements will then be translated to circuitry that can be manufactured on a PCB (Printed Circuit Board). This includes choosing ICs (Integrated Circuits), components, and component values based on the technical literature and experimentation. This will form the requirements of the analog computer such as components, size, and complexity.

The functionality of the artifact will be tested with a demonstration of its ability to solve specific differential equations. These differential equations are the spring-mass dampening system and Mathieu's differential equation. A simple oscillator will also be used to evaluate the computational elements. The result of which will be compared to circuitry specifically developed to solve these two equations. The larger role of the artifact and analog computing in a digital society will also be investigated based on the empirical material provided in this paper. The steps this paper will take in order to develop the artifact is:

- Investigate the different computational elements of an analog computer.
- Translate the computational elements to circuitry.
- Develop the PCB that will make out the analog computer.
- Evaluate the functionality by demonstrating the predetermined differential equations and compare the result of the analog computer with the custom circuitry.
- Present the results found in the evaluation in the context of an ever growing digital society with a focus on education and practical application of analog/hybrid technology.

VI. TERMINOLOGY

A. *Analog computer*

Analog is derived from the Greek words ana-logon. The translation is interpreted as "according to a ratio". This could be explained as a connection between different relationships in regards to material and constructs. A sail catching the wind, an airplane wing creating lifting force, and the submarine deep in the Atlantic ocean are all different constructs but analogous in the way they deal with physical forces.

The electric analog computer's output from operational amplifiers is a response to its input signals. The output varies depending on the input. Utilizing this, a system that can analogously respond to different conditions can be modeled to simulate different real-world phenomena [22].

Analog computers were and still are based on variable voltages to calculate and represent the solution to a given problem. Analog computers handle computation by utilizing logic built by different operational amplifier-based circuits for different mathematical calculations based on variable voltages in the circuit [5].

The analog computer is highly suited for calculating second order differential equations. The problem the user wants to solve (by the equation) is first translated into an equation. The equation is then rearranged, in this example using the Kelvin feedback technique [5], where the highest derivative is isolated on the left side of the equation. By then feeding this derivative into a specific chain of computational elements (integrators, summers, etc.) the right side of the equation will be obtained.

By creating an analog system, a physical system is configured and translated to mathematical abstraction. By introducing an initial condition that corresponds with the problem's initial value the analog computer will simulate the problem over time starting at T_0 to T_f (meaning the start and stop of the simulation) and present the results continuously [8] [5].

Digital computers rely on relays representing ON (1) and OFF (0) to calculate and display the solution to a problem. This is called boolean algebra and is the norm for digital computers today.

B. Boolean algebra and discrete mathematics in computer science

Boolean algebra is the logical language where you are able to express any statement through a binary form [23]. In computer science this often refers to machine language, ones and zeros or on and off pulses. Due to the vast versatility of the different logic gates that can be created by combining transistors boolean algebra and discrete mathematics quickly rose as the leading language of computer science [24].

There are many advantages with discrete mathematics and boolean algebra when it comes to computer science, mainly giving in this case the programmer the ability to be very precise when running calculations. It takes a lot for a one to become a zero in comparison with a 1,00001 becoming a 1,00002 in the field of continuous mathematics [25].

Despite being the computer language of today, discrete mathematics and boolean algebra has a big bottleneck when it comes to its use in computer science. This disadvantage stems from the lack of variability in the throughput which forces digital computers to work sequentially. A digital computer can only process either a one or a zero at a time which limits the computational speed to the rate in which you can switch between a one and a zero [25].

C. Differential equations

As Braun and Golubitsky state in there book *"Differential equations and their applications"* differential equations occur in many areas of science [26]. In this section we will briefly explain what a first and second-order linear differential equation is and give some examples of what they are used for. Important to understand is that a derivative can be interpreted as a rate of change seen over time. The first derivative is the rate of change and the second derivative is the acceleration of change.

1) First order linear differential equations: A first order differential equation is defined by the equation below:

$$F(t, y, \dot{y}) = 0$$

In other words, an equation consisting of the unknown function y and its first derivative $\dot{y}$ with respect to time t. First order differential equations occur in a lot of different fields of science, a few examples of those are:

- Newton's law of cooling:

$$\dot{y} = k(M - y)$$

- Eikonal equation:

$$H(x, \nabla u(x)) = 0$$

- Radioactive decay:

$$-\frac{\mathrm{d}N}{N} = \lambda \mathrm{d}t$$

2) Second order linear differential equations: A second order differential equation is defined by the equation below:

$$F(t, y, \dot{y}, \ddot{y}) = 0$$

This is essentially the same as a first order differential equation, except that y second derivative $\ddot{y}$ also is included.

D. Simple oscillator circuit

The oscillator circuit is described in Ullmann's book *"Analog and Hybrid Computer Programming"* [5] the equation for the oscillator goes as follows:

$$\ddot{y} = -\omega^2 y$$

where ω^2 is some arbitrary weight. The logic schematic is:

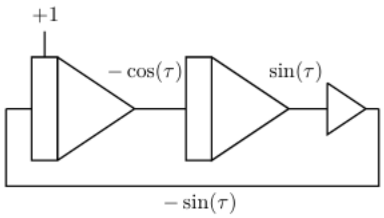


Fig. 2. Simple oscillator [5]

The oscillator can be used as an effective tool to test the functionality of analog computers (integrators, invertors and summers). It was used in this project as the first functionality test and the output (solution) should be displayed as a steady waveform.

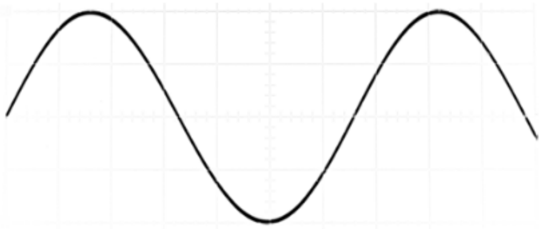


Fig. 3. Simple oscillator output [5]

E. Mathieu's differential equation

Mathieu's differential equation is a well known, linear homogeneous, second order differential equation. The equation has many different applications ranging from the vibrations of an elliptic drum to balancing an inverted pendulum [27].

As described by Ulmann [27], the general expression of Mathieu's differential equation is:

$$\ddot{y} + (a - 2qcos(2t))y = 0$$

With the initial conditions:

$$y_0 = 1$$

$$\dot{y}_0 = 0$$

a and q are parameters that has been assigned a value based on the given problem. To simplify the calculations one can set $a = 2q$. By doing this one gets the calculation:

$$\ddot{y} + (a - acos(2t))y = 0$$

One can then factor out a out of the parentheses which results in:

$$\ddot{y} + a(1 - cos(2t))y = 0$$

The resulting equation is now dependent on y (the sought after value) and t (the solution to the equation when taking time into account a is still a parameter one can assign any value to. The parts that is dependent on t can be seen as an "input signal" that can be modeled. Therefore one can call it $x(t)$ or just x. To ease the calculations we introduce the x variable to the aforementioned equation by the following statements:

$$x(t) = 1 - cos(2t)$$

$$\ddot{y} + axy = 0$$

Based on the aforementioned explanation, these statements holds true.

The general expression for y can be exemplified by analog computer schematics, see fig 4.

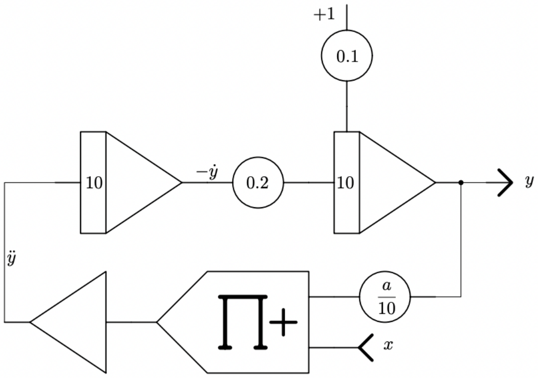


Fig. 4. Y for Mathieu's equation [27]

The general expression for x in the Mathieu's differential equation (5) can be expressed as:

$$\ddot{x} + 4x = 4$$

Analog computers are ideally suited for solving differential equations. To specify what differential equation to use, the Mathieu's differential equation was chosen because of its prevalence in academic resources.

The analog computers schematic for solving Mathieu's differential equation can bee seen in fig 4 and 5. Typical solutions for Mathieu's differential equation can be seen in fig 6

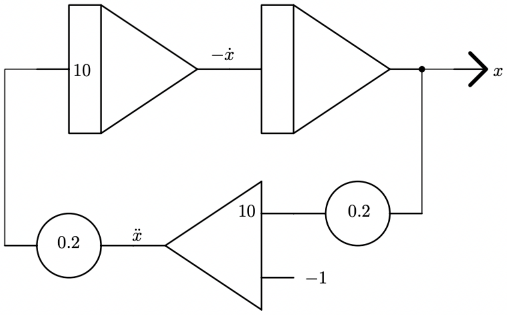


Fig. 5. X for Mathieu's equation [27]

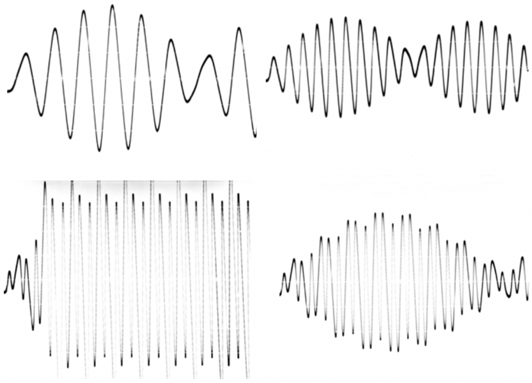


Fig. 6. Typical solutions for Mathieu's differential equation [27]

F. Spring mass dampening system

The spring mass dampening system is a good example of an oscillator with dampening. This simulation is an analog of a weight suspended on a spring with the mass m. The oscillation is the weight bouncing up and down with the position y, thus the vertical position. According to ´ Ulmann [5], to simplify the calculations, the gravitational forces acting upon the weight will be neglected. The force due to the moving mass (weight) can be expressed as:

$$F_m = ma = m\ddot{y}$$

The spring will introduce a force depending on the strain applied to it (pulling the string) and the force of the linear velocity dampening can be expressed as:

$$F_s = sy$$

$$F_d = dv = d\dot{y}$$

The spring mass dampening analog is a closed physical system. This means that all the forces in the system needs to add up to 0. The second order differential equation based on this information can be expressed as:

$$m\ddot{y} + d\dot{y} + sy = 0$$

The m is the mass, d is the dampening constant and s is the spring constant. Important to note, with out F_d no dampening will occur.

The actual equation the analog computer can solve, and has the possibility to be easily converted to a schematic, is:

$$\ddot{y} = -\frac{d\ddot{y} + sy}{m}$$

The schematic that will be used in this paper to create a circuit is a simplified version in order to save one summer. The disadvantage of this approach is that constants s and d can not be set independently. The simplified approach has s and d as a fraction of m. The simplified version introduces two new parameters called v and μ. The mathematical expression for these two are:

$$v = \frac{s}{m}$$

$$\mu = \frac{d}{m \cdot}$$

The two potentiometers $-\dot{y}0$ and $y0$ are used to set the initial condition of this analog. $y0$ sets the initial deflection and $-\dot{y}0$ sets the initial velocity of the mass which is set to 1. Different values for s and d will result in different results as the spring and dampening will be seen in the oscillation of the mass. The analog computer schematic can be seen in fig 7 and typical solutions can be seen in fig 8.

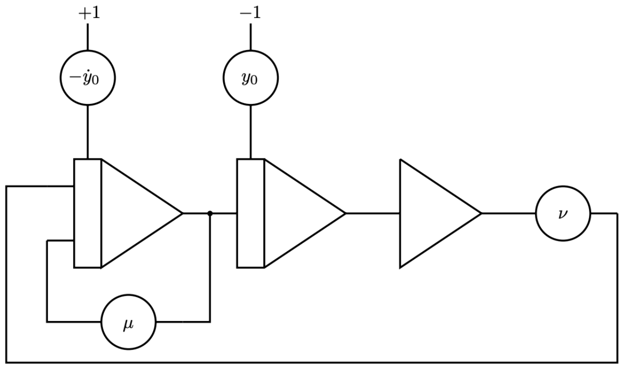


Fig. 7. Simplified spring mass dampening [5]

VII. RELATED WORK

A. The computational advantage of analog

The analog computer does not run on a sequential basis like its digital counterpart. The added flexibility of the digital computer is due to the ability to run it on a joined clock system (enabling time sync between two digital computers enabling precise simultaneous operation) and being able to solve a variety of tasks due to the aforementioned sequential algorithm structure. The main problem with the current development of the digital computer is that the digital computer is rapidly approaching Moore's law, thus hindering further progress [2] [5].

Fig. 8. Typical solutions for spring mass dampening. Top graph $v = 0.6$ $\mu = 0.8$ Bottom graph $v = 0.8$ $\mu = 0.6$ [5]

The comparison between analog and digital is something academia has covered in regards to time, the complexity of problem, and energy usage. A paper written by Köppel et al [2] uses a common differential equation. Their experiment showed that the analog computer did not take longer to compute given the complexity of the problem but the digital computer did. The energy consumption did also increase for the digital computer (which was significantly higher initially) when the complexity increased, whereas the analog computer remained at low levels of energy consumption throughout the experiment.

This is due to the fact that the analog computer does not run on a sequential boolean algorithm. Instead, the Analog computer runs continuously and directly using the op-amp (operational amplifier) based computational elements. This is the main advantage of using an analog computer according to the writer.

The common digital computer is a so-called a "stored program computer" and has a fixed internal structure. This fixed internal structure is then controlled by an algorithm which is stored as a program in memory inside the machine. This structure has the tendency to create data dependency and memory bottlenecks that impact the overall performance negatively. The analog computer is in comparison not reliant on memory or having a fixed inner structure. The different computational components are arranged in a way to create an analog of the mathematical problem that is being simulated. This speeds up the process of computing and simulation of a given task due to all calculations running in parallelism to each other [28] [2].

In a paper written by Holzer and Ulmann [28] they discussed the integration of analog computing when working with machine learning (ML). They created a hybrid system where the analog computer was closely connected to a digital machine. The analog computer was continuously simulating the balancing of the inverse pendulum and the ML algorithm used this information as data for reinforced learning.

The analog computer was utilized as a powerful co-processing unit to solve the differential equation-based problem of balancing an inverted pendulum. The reinforcement learning was done episodically where one episode is from the start to the pendulum tipping over or the cart moving outside the designated area. The digital computer asked the analog for the real-time simulated values, the communication went through a hybrid controller (by serial communication) that converted the information between the two systems. The digital computer, running the ML algorithm, then decided if the episode was to be terminated and reset or continued. If the pendulum was mounted on a fixed cart the equation would have been:

$$\ddot{\varphi} - \frac{g}{l} sin(\varphi) = 0$$

Since the cart was not fixed, the mathematical expression was greatly more complex than if the cart would have been stationary. The authors goes great lengths proving and simplifying the differential equation actually being calculated by the analog computer. The differential equation that was being solved was:

$$\ddot{\varphi} = \ddot{x} cos(\varphi) + g sin(\varphi)$$

And the acceleration of the cart can me modeled as:

$$M\ddot{x} = F$$

The length of the pendulum was assumed to have the length 1 ($l = 1$) and is therefore removed in the above equation. The φ is the angle between the the pendulum and the vertical axis (y). g is the gravitational acceleration, x is the position along the horizontal axis (x), M is the mass of the cart and F is the force applied to the cart to stabilise the pendulum.

The results of the study by Holzer and Ulmann [28] showed that a hybrid system is possible when dealing with ML and that having an analog computer simulation the problem was more stable and more energy efficient than a purely digital system. The paper also suggest that great computational improvements could be achieved if there was an integrated analog co-processor embedded in modern digital systems, thus increasing computational and simulation capabilities.

B. Analog computers in academia

Since the invention of the general-purpose electronic analog computer the usage in engineering and physics laboratories has been prevalent. Given the nature of the analog computer and its ability to solve differential equations efficiently, the analytical solution to problems was not the default approach. The analog computer made the numerical approach possible as the approximation needed was easily introduced in programming. In the early days of analog computing, the experimentation process was greatly sped up by having the possibility to easily program (compared to its contemporaries) a problem and see the results almost immediately. An example could be to change coefficient values, which required lengthy manual calculations through the analytical approach, which could now easily be introduced by turning a potentiometer (or changing resistor depending on the set up) [29].

The usage of electronic analog computers can also be applied to academic endeavors in other ways than simulating physics or engineering problems, such as a pedagogical instrument (see fig 9). Students can be enrolled in mathematics classes without ever visualizing a differential equation. Students are thought to instead create an expression that satisfies the equation through a systematical (perhaps even mechanical) approach that gives little deep understanding of the process. The implementation of analog computers in this context is both cost-effective and shows a positive impact on the learning outcomes due to an increase in intuitive calculations by the students. The analog computer allowed the students to work on nonlinear and linear systems while emphasizing the relationship between the mathematical and physical models. This meant that there was a knowledge combining element between the math and physics-focused classes [30] [31] [32].

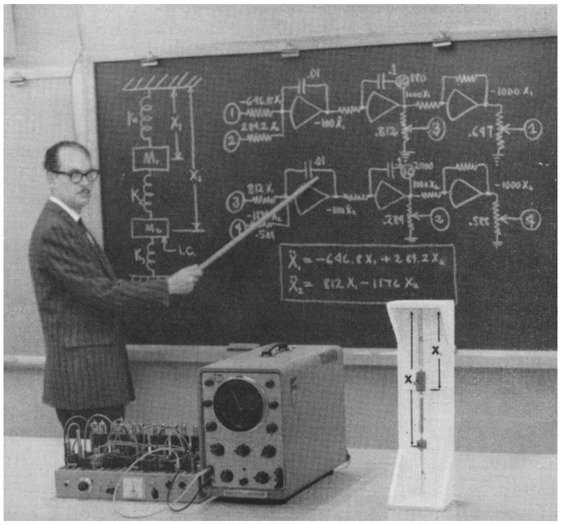


Fig. 9. A demonstration of the solution to a differential equation based problem using an analog computer [30]

A low-cost, desk-sized, analog computer for use in academia has been of interest to technical faculties in the past. Hamilton [33] proposes a design for a machine (see appendix E for schematic) in a paper called "An Analog Computer for Educational Laboratories". The machine is a general-purpose analog computer that features several computational elements such as integrators, summers, dividers, and coefficient potentiometers. The design also features an overload detection, meaning when an overload of the operational amplifiers occurs.

An example of a desk-sized analog computer that saw a long life in academia is the Comdyna GP-6 released in 1966 [34]. The GP-6 longevity in academic institutions can be explained by its use in control unit education. The example given in Spiess [35] paper describes the students acquiring more knowledge of electronics and the connection to mathematics while using an analog computer. The computer used in the paper is a GP-6 and is a staple in the control systems laboratory at the University of Illinois. Programming an analog computer requires an understanding of the fundamental building block of the analog computer. This includes a deep knowledge of the operational amplifier and its application of it in summers, integrators, multipliers, and dividers. The students are also forced into understanding the qualities of high gain, high input, negative feedback, and low output impedance and applying Kirchhoff's Laws. The author argues that the best way for students to learn about linear circuits and their different components is to force them to create one by using an analog computer.

C. Modern analog computer

A new research initiative called The Analog Paradigm [36] is currently being developed to inform and create analog and hybrid systems that can be used in today's digital world. The Analog Paradigm has developed an low cost, scalable analog computer that is able to be both daisy chained with other analog computers and be converted to a hybrid computer through the usage of a single board computer.

The analog computer The Analog Paradigm are developing is called The Analog Thing (THAT) [37] and will be released in Q2 of 2022. THAT used to calculate mathematical problems and simulate real world mathematical problems. Programming is done by utilizing the patch panel located on the device (see fig 10).

Fig. 10. The Analog Thing [37]

The core concept of programming THAT is to translate the change in a dynamic system into a mathematical problem, often a differential equation. This equation is then set up by the wiring up the patch panel to represent the equation. The result of this programmed differential equation is then presented as a time varying voltage.

To use THAT in a hybrid system (with a single board computer) one has to utilize several DAC/ADC (analog to digital/digital to analog converters) in order for the two systems to communicate between each other. The communication is handled by connecting the two machines together with a cable and through an Arduino shield (see fig 11).

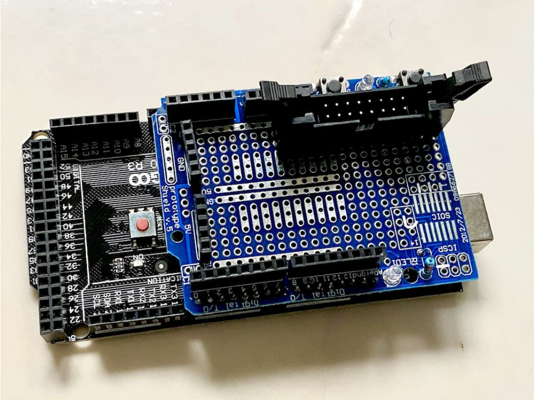


Fig. 11. THAT Arduino shield [37]

By connecting THAT to the Arduino a hybrid system is created. The analog computer can then be used as a powerful co-processing unit for the Arduino. This gives the Arduino more capabilities in terms of computational power and speed to solve a specific problem [37].

The artifact THAT is more than just a product. The analog paradigm offers an artifact that is connected to a larger platform for analog computing. The THAT wiki is a collection of information and a project that aims to spread the knowledge of analog computing to a wider audience. The platform is still in its infancy but is reminiscent of the Arduino online platform with a similar purpose. One could argue that the THAT artifact has the same strengths as Arduino according to the definition of Bjögvinsson et al. [38] designing Things. Things are an expansion of the view of what an artifact is and should be. Bjögvinsson et al. argues that artifacts should become a part of socio-material assemblies, being socio-material frames for controversies, opening up new ways of thinking and behaving as well as being ready for unexpected use. This view of artifacts can also be seen in the Arduino platform and could partly be a reason for its success [39].

D. Modern applications of analog computers

Analog computers have also seen a resurgence in research related to artificial intelligence (AI) and neural networks. The neural networks being created are getting closer to how a human brain works. The human brain and computation surrounding it have traditionally operated under the assumption that the brain operates on an all-or-nothing basis. This would make the contemporary digital computer possible to operate as an analog, given the fact that it operates in a similar boolean fashion. This could stem from the fact that the most well-developed accounts of computation are based on digital computation, leading to a bias in modern computation.

New discoveries within the field of neuroscience strongly suggest that the brain does not operate in a boolean fashion. Maley [40] argues that the brain does operate like a computer, but only if one expands the view of what a computer actually is to include the analog computer. Analog computation is a significant chapter in the history of computation and deserves to be acknowledged as such. The inclusion of analog computation can therefore not be seen as an improvised inclusion due to the historical prevalence of the computational paradigm.

The former established view on neural activity is that neural spikes (firing) are the basic building block of neural signaling [41]. Findings over the last decades have found that a neuron has to reach a certain voltage threshold for a neural spike (firing) to be created. This is where the notion of the close relationship between the digital paradigm and neuroscience stems from. A parallel could be that if a certain voltage threshold is reached in a digital computer it signals "ON" (1 if one uses the 1 or 0 examples). The digital computer does not signal any variety of "ON" if a higher/lower voltage is present.

The problem with looking at neural activity in this manner is that the action potential is not taken into account. Depending on if a neuron fires, the voltage traveling down the axon will, at the end of the axon, release neurotransmitters at synapses where the axon is close to other neurons. The neurotransmitters will then act as an input for other neurons further down the chain. This input can affect the voltage of the next neuron (increase or decrease) thus impacting its likeliness to fire, thus establishing the action potential.

Although a lot of the communication is based on action potentials some rely on direct electrical signaling across synapses. This is called a gap junction and are continues and does not follow a chain as the previous example did. The current flows seemingly unchanged between the neurons and is there for best explained as not being gated or attenuated. The impact of these gap junctions has previously been stated as being insignificant but is now considered underestimated in their impact on the way the brain works. This continues operation is therefore a consideration to include analog computation in the definition of how the brain computes [40][42] [43].

The current hardware being used when developing and working with deep learning algorithms and AI is graphical processing units (GPU). Since the mathematical operations required for deep learning are limited and the GPU can easily be parallelized and configured it has been the standard in today's digital environment. The origin and popularization of this approach stem from the game and visual effects industry for usage in 3-D rendering especially [44]. Similar to a human brain, deep learning and deep neural networks are composed of layers of artificial neurons. Each neuron derives the output of those in the next layer. Based on a pair of values, the neuron activates and the synaptic "weight" connects to the next neuron in the chain [45]. This is very similar to the description of the relationship between neurons, axons, neurotransmitters, synapses and the traveling voltage in the human brain explained earlier by Maley [40].

Deep learning and deep neural networks (DNN) are often slow (as in timely to produce a result) or very energy demanding. They can even be both depending on the complexity of the task. One of the largest factors contributing to this problem is the nature of how the digital computer operates. When data is being moved from external memory and computational resource (CPU or GPU) the architecture that separates memory and logic limits the speed. This phenomenon is called the von Neumann bottleneck and is a known source of inefficiency [46]. Most meaningful neural networks are too large to store within processor memory. This means that weights must be brought in from external memory as each layer of the network is computed, thus the von Neumann bottleneck takes full effect. The conventional approach to limit this problem is to use DNNs that move fewer weights in from memory, which the digital computer favors, then aggressively reuse these weights [45].

Analog AI and the technology behind creating it (mostly related to chip technology) was hypothesized at IBM research in 2014. The technology IBM referred to as Analog AI was also referred to as crossbar arrays. One could describe it as where devices, memory cells, for example, are built in the vertical space between two perpendicular sets of horizontal conductors, the so-called bitlines and the word lines. Other names for this phenomenon can be in-process memory or computational memory [45] [47].

The solutions to in-process memory for deep learning are analog in the sense that they do not work as a conventional digital system. The IBM crossbar arrays are just one example of an analog solution. Other examples might include, ferroelectric devices (a stack of thin dielectric and ferroelectric materials located between a conductive electrode and a substrate), electrochemical devices (A signal is run over a solution that changes the resistance between two electrodes due to electrons moving between the electrolyte and host matrix, thus working somewhat similar to a battery) and restive Random-access memory (a metal oxide layer sandwiched between metal contacts which are then infused with oxygen to affect restiveness. The top contact controls the infusion of oxygen that forms a conductive filament of metal oxide)[44].

The aforementioned examples are as of yet only present in the research domain. They are not flexible in their approach and require a digital computer in order to be used in a wider array of tasks. When looking at an analog computer, the ability to reprogram and wider use cases becomes self-evident. The analog computers' ability to solve linear differential equations is one of its core strengths. The usages for this could therefore be in analog co-processor applications. An aforementioned example could be the reinforcement learning of an inverted pendulum [28]. Cowan et al. [48] describes a Very-Large-Scale-Integrated Analog Computer (VLSI) in a paper called "A VLSI analog computer/digital computer accelerator", witch is essentially a chip version of an analog computer. The chip is powerful as it delivers 21 gigaflops per watt for certain differential equations [1]. The paper describes that the analog chip dissipates only 0.02% to 1% of the energy a general-purpose digital microprocessor would have and about 2% to 20% of the energy of a digital signal processor would have. Thus proving it is highly energy-efficient compared to digital contemporaries. The efficiency is so high it even rivals most inclusions on the Green500-list [1].

The uses of the analog computer covers more than just the linear homo genus differential equations previously mentioned. The analog computer can also solve ordinary differential equations as well as partial differential equations. This includes the Lorentz system (attracter) that simulates a chaotic system and the heat wave equation. A problem with solving these types of equations is that working with problems with respect to more than one variable is not self-evident. This is because the analog computer generally only allows integration with respect to machine time (the internal time the machine integrates at). The way to handle these types of equations on an analog computer is to approximate all differentials which are not with respect to time as differential quotients, thus discretizing the underlying problem with respect to the resolution [5]. This can also be explained as simplifying the problem to only be dependent on the time variable.

The introduction of faster wireless communication (5G) promises an increased capability of IoT networks. The faster speed at which communication can travel increases the need for faster computation of IoT devices [49]. The faster computation needed has the side effect that a bottleneck of information can occur based due to the digital processor used not being fast enough [50]. An example could be an IoT-connected drone that needs to compute real-time positioning data in the 1-2ms range (even faster in some cases). This is not a simple task using digital technology [51] that requires substantial energy consumption and expensive hardware. The same example holds true for self-driving cars where 40% of the total energy consumption is by the digital computer [52]. This is an application where analog technology could substantially improve energy consumption and computational speed. The computational speed advantage could also be applied to balancing a ball inside a magnetic field using two electromagnets as Huang et al. [53] describes. The digital solution for solving this problem is also not trivial. Analog technology could be used as a co-processor to speed up the calculations to increase the stability of the system [53]. Analog magnetic field control calculation is also something that can be implemented in systems needing even faster computation, such as particle accelerators. A notable system using magnetic field technology is the ESS system located in Lund, Sweden [54].

E. The different components of the analog computer

A generic Analog computer offers several different computational elements (operational amplifiers, summers, integrators, coefficient potentiometers and multipliers). These different computational elements can be freely connected through a patch panel.

Depending on how one connects these different computational elements different mathematical calculations can be set up. The analog computers programming is often depicted as high level analog computational component schematics. This means that the actual circuits are not represented as a schematic. This section will present the different components and their high level component symbol used in this paper.

F. Inverting and non-inverting operational amplifier

The inverting op amp depicted in fig 14 has two resistors, R and R_f. These two resistors decide the weight factor described before. A typical weight factor for an analog computer is either 1 or 10. The formula for the weight factor can be expressed as:

$$a_i = \frac{R_f}{R}$$

Fig 12 depicts a very simple example of an inverting operational amplifier. The amplifier only has one input e_i and is connected to the output e_o by the resistor R_f.

The operational amplifier (U1 in fig 12) has two inputs. One is inverting (marked by the - sign) and one is non-inverting (marked by the + sign). If the circuit is configured as fig 12 depicts, it is an inverting op amp. To calculate the output of the op amp the formula is:

$$e_o = -\frac{R_f}{R} e_i$$

The result of the amplification is expressed as e_o and is the amplified and inverse voltage compared to the e_i [5]

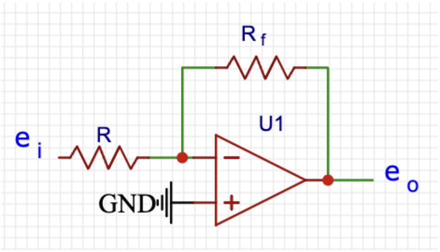


Fig. 12. Inverting op amp

All the inputs to the operational amplifier, and the feedback path, is connected to the inverting input. This is called a summing junction (SJ) and computational elements like summers, and integrators often makes this available to the user. This means that additional feedback circuits or input resistors ect. can be connected to the computational element [5].

The non-inverting operational amplifier [55] (fig 13) has the same characteristics as the inverting op amp. The core difference is that the non-inverting op amp will not create the inverse voltage thus the same polarity will be on both e_i and e_o. Note that the non-inverting side (+) is used in this circuit. The output from this circuit e_o can be expressed as [5]:

$$e_o = \frac{R_f}{R_i} e_i$$

G. Summer

The summer (fig 15) consists of an op amp that allows for different currents to be summarized. The currents are e_i and are connected by a resistor R_i to the inverting side of the op-amp (to SJ). The feedback resistor connects to the inverting input of the op-amp via a feed back resistor R_f to the output e_o. The weights of the different inputs are described by a_i and are often 1 or 10. The weight ratio is described by $a_i = \frac{R_f}{R_i}$

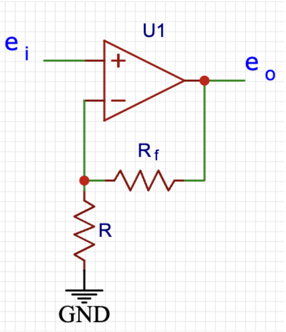


Fig. 13. Non-inverting op amp

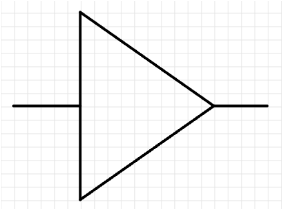


Fig. 14. Operational amplifier symbol [5] [56]

(as previously described for the operational amplifier) and is often depicted in symbol based schematics.

The formula for the inverting summers (fig 16) output voltage e_o where R_f depicts the different input resistors to SJ and e_i being their incoming voltage [57] [58] is:

$$e_o = -\frac{R_f}{R_i} \cdot e_i$$

To further exemplify one can also express it as [58]:

$$e_o = -\frac{R_f}{R_i}(e_1 + e_2 + e_3 + e_4...)$$

The problem with that is that the weight factor is not taken into account in the above expressions. The complete mathematical expression that takes the weight factor into account is [58]:

$$e_o = -R_f(\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} + \frac{e_4}{R_4} + ...)$$

Which can also be expressed as [5]:

$$e_o = -\sum_{i=1}^{n} a_i e_i$$

Here n is the number of input resistors.

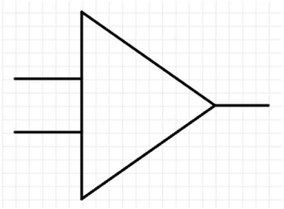


Fig. 15. Summer symbol [5] [56]

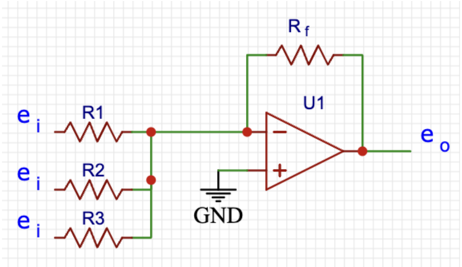


Fig. 16. Summer circuit

H. Integrator

The integrator is the always operating in respect to time in the electro analog computer. There for the initial condition (IC) can be seen as the definitive start of the integrator operation (depicted as IC in fig 17 and $e(0)$ in fig 18), this introduces the initial values for computation. The general expression all the integrations follows in an analog computer is:

$$e_o = -(\int_0^t \sum_{i=1}^{n} a_i e_i dr + e(o))$$

The relationship between R and C (resistor and capacitor, R1 and C1 in fig 17) is determining the time scale factor($k_0 = 10^n$) of the integrator and n typically ranges from 1-3 in larger machines [5] [57]. By changing the time scale factor one can effectively control the speed in which the integrator computes. By either speeding up or slowing down the time factor the operator can time scale the problem being investigated. An important aspect is that the problem time (the time scale the problem runs at) and the machine time (the time the machine runs at) is not necessarily the same and is determined by the time scale factor of the integrators. The inputs of an integrator can be weighted much like a summer. The implementation of the integrator is very much like the summer as well but the difference is that the feedback resistor (R_f) is replaced with a capacitor (C). Important to note is that the switches controlling the operation of the integrator should work in total harmony without contact bounce in high precision machines. Analog relays could work in less complex, slow, analog computers but could have a detrimental impact in calculations. MOS based switching is ideal if implemented correctly.

The calculation for the weight factor for the integrator is [5]:

$$a_i = \frac{C}{R_i}$$

The time factor is usually 1, 10 or 100. The time the integrator computes can be calculated by:

$$T = RC$$

There T the charge and discharge cycle of the capacitor in seconds. An example of different time factors could be to have a 100k ($10^5 ohm$) resistor and different values of capacitors. A 1uf capacitor is 10^{-6}f, the calculation for the time factor would be:

$$10^5 * 10^{-6} = 0.1s = 100ms$$

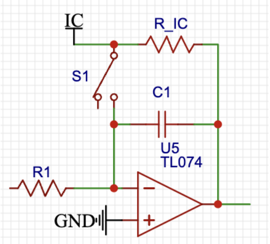


Fig. 17. integrator circuit with IC

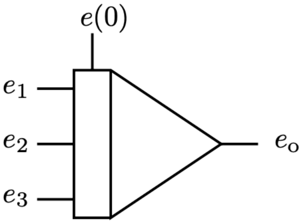


Fig. 18. integrator symbol [5] [56]

I. Potentiometer

The analog computer uses potentiometers 20 to set different coefficients according to the users specifications [5]. The most basic use of the potentiometer in analog computing is as a voltage divider. The voltage divider ranges from 0 to 1 (machine units in analog computer terms) where 0 means fully closed potentiometer (tying signal to ground) and 1 is fully open potentiometer. The voltage divider setup can be seen as upper resistor R_u and a lower resistor R_l. The relationship between the two resistors is decided by turning the position of the potentiometer knob a. The formula of the relationship can be described as:

$$R_u = (1 - a)R$$

$$R_l = aR$$

This means that setting $a = 0$ (turning the potentiometer fully closed) effectively ties the output directly to ground, therefore terminating the signal. The overall resistance of the potentiometer R can be described as an addition of the upper and lower resistor:

$$R_l + R_u = R$$

Since the potentiometer acts as a voltage divider, the expression for the output voltage e_0 (e_i being the input voltage) can be expressed as [5]:

$$\frac{e_o}{e_i} = \frac{R_l}{R}$$

This means that the relationship between R_l and R ranges between 0 and 1. This is often depicted in high level analog schematics to represent what the potentiometer should be set at. See fig 19 20 for schematic and analog computer schematic symbol.

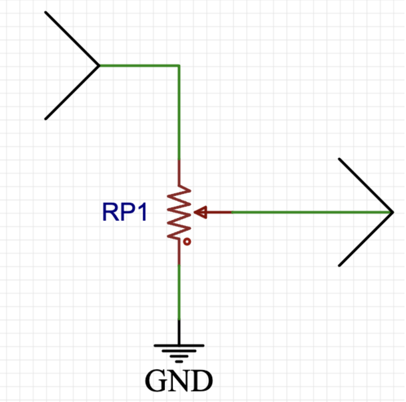


Fig. 19. Potentiometer circut

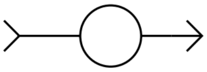


Fig. 20. Potentiometer symbol[5] [56]

J. Multiplier

Multiplication using analog technology is not as simple as one might assume. It is one of the more complex computational elements in an analog computer. A lot of ingenious methods of analog multiplication has been developed over the years but the most common today is the Gilbert cell multiplier due to its high precision and ease of integration into an IC. Modern analog multipliers only needs two inputs in order to work, x and y ($x \cdot y$) [5].

The analog multiplier that will be used in this paper is the AD663 from Analog Devices. It is a four quadrant Gilbert cell multiplier with high impedance, differential x and y (x, $-x$, y, $-y$) inputs, and a high impedance summing input z [59].

The most common symbol for the multiplier can be seen in fig 21. If a multiplier does sign inversion the symbol needs to reflect this, see fig 22. Sign inversion is done by connecting to the inverse input of the differential x and y of the multiplier [5].

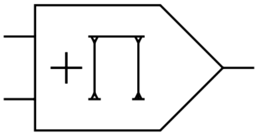


Fig. 21. Multiplier symbol[5] [56]

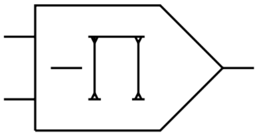


Fig. 22. Multiplier with sign-inversion symbol[5] [56]

VIII. RESULTS

A. Circuit design

In order to better understand the actual circuits of an analog computer some of the high level symbols presented in [5] has been translated into common circuits. The whole circuit design started by experimenting with a breadboard and through-hole components to make it easier to see what works in the real world as apart from what works purely theoretically. This is a more experimental approach and has the advantage of being more flexible and produce results directly, compared to create PCBs and solder the components to it initially.

B. Breadboard experimentation - Spring mass dampening

During the translation of the schematic the two integrators had the time factor of 1 according to fig 17. To achieve this 10nf capacitors and 100k resistors were used. The time factor calculation for this circuit is:

$$T = RC$$

$$10^5 \cdot 10^{-8} = 0.001s = 1ms$$

During experimentation, a capacitor value of 1uf resulted in a visually more real world (real time) analog. The time factor calculation for this would be:

$$10^5 \cdot 10^{-6} = 0.1s = 100ms$$

If time factor 1 would be 1ms (10nf), the time factor of the 1uf capacitor integrator would be 0.01 using the same scale and would run at 100ms (since 100ms is 100 times slower than 1ms). For the complete schematic see fig 23 and for the analog computer schematic see fig 24.

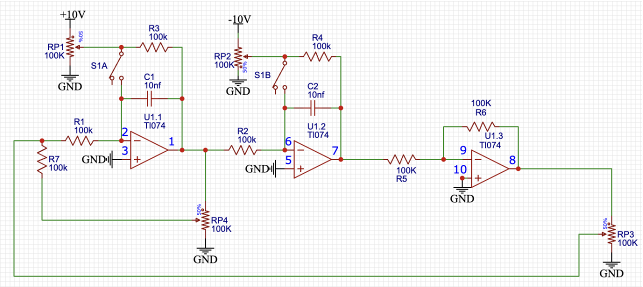


Fig. 23. Schematic Spring mass dampening

The two potentiometers $-\dot{y}0$ and $y0$ (fig 24) are labeled $RP1$ $RP2$ in the schematic (fig 23) and set the initial condition (IC) of the two integrators. The two potentiometers in fig 24 labeled μ and v are $RP3$ and $RP4$ in fig 23. As mentioned before, they set the spring and dampening coefficients for the analog. They are dependent on the mass according to:

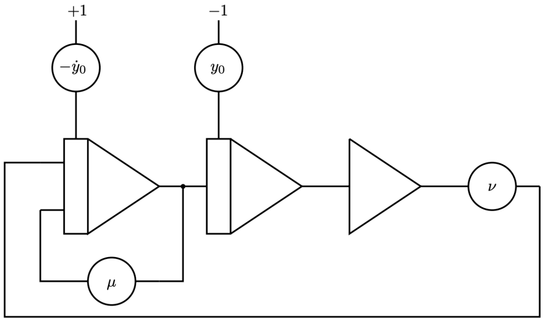


Fig. 24. Schematic Spring mass dampening

$$v = \frac{s}{m}$$

$$\mu = \frac{d}{m \cdot}$$

The readout on the oscilloscope is depicted in fig 25.

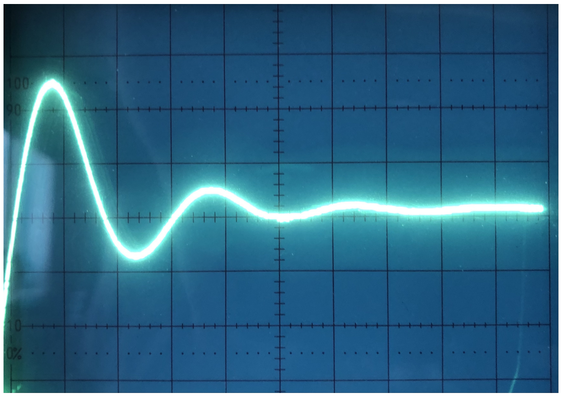


Fig. 25. Ocilloscope read out with $\mu \approx .7$ and $v \approx .6$

C. Breadboard experimentation - Mathieu's equation

The circuit in fig 5 and fig 4 was analysed and translated into a schematic (fig 26) that resembles a more conventional schematic compared to the abstractions that Ulmann [27] describes. An example of the output from the circuit can be seen in fig 27

The schematic in fig 5 for the X variable of Mathieu's equation consists of a loop containing a summer, two coefficient potentiometers and two integrators. If we assume the starting point is at the coefficient potentiometer at the bottom right, the first summer is weighted with a factor of 10 at the first input. This is achieved by having the input resistor value 10k (R2) and R_f (R1) (as described previously) at 100k. The second input is not weighted and has a 100k resistor (R9) as an input resistor, the input is the full negative voltage of the circuit as described by -1. The output of the summer ($\ddot{x}$) is then passed

passed through a coefficient potentiometer with the value of 100k (RP2). The first integrator has a weighted input of 10, to achieve this, a 10k resistor is used as an input resistor and a 1nf capacitor (C1). The output if the first integrator ($-\dot{x}$) is then passed to the second integrator. The second integrator is not weighted, therefore a 100k input resistor (R4) and a 10nf capacitor (C2) is used. The output from this integrator (x) is then looped back via a coefficient potentiometer valued at 100k (RP1) to the summers weighted input.

For the y variable of the Mathieu's equation see fig 4. The x variable is connected to the non-inverting side of one of the factor-inputs of the AD633 analog multiplier. The inverting side of the same factor is connected to ground. The output of the AD633 analog multiplier is connected to an inverter which is non weighted. The input resistor is valued at 100k (R5) and R_f (R6) is also valued at 100k. The output of the inverter $\ddot{y}$ is connected to the first integrator. The integrator is weighted by 10, meaning that the input resistor is 100k and C3 is 1nf. The output of the first integrator goes through a coefficient potentiometer to the second integrator. The last integrator is also weighted by 10 thus having the same configuration (C4) as the previous integrator. In addition to this a OP-mode switch controlling the operation of the circuit (SW1) is added. The integrator also has a potentiometer that introduces the IC to the integrator (RP5) controlling the positive full voltage, +1, of the IC. The IC also consist of a switch (SW2) to introduce it to the circuit as well as a discharge resistor (R8). The output y is then looped back the the non inverse side of the other factor side of the AD633 analog multiplier. The inverting side is connected to ground.

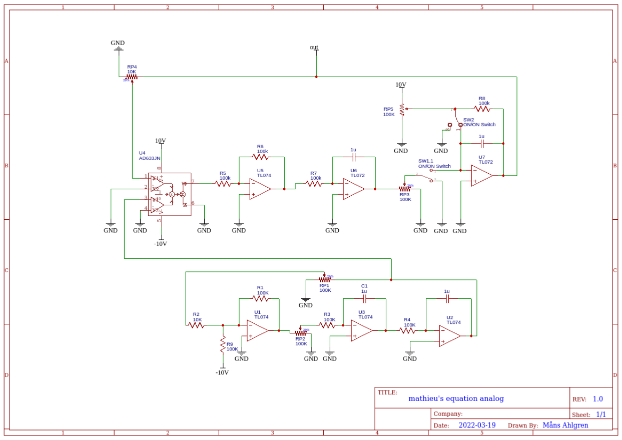


Fig. 26. Schematic Mathieu's equation

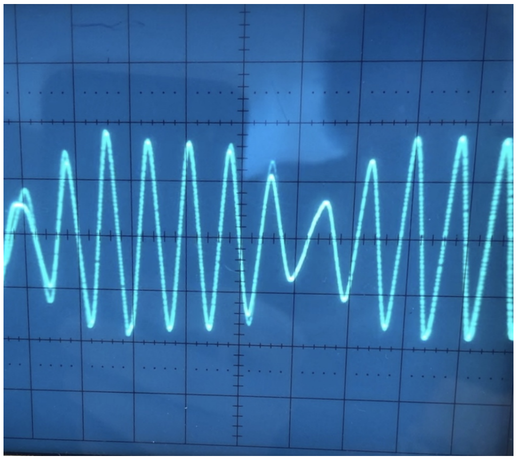


Fig. 27. Ocilloscope readout of Mathieu's equation

D. V1 of the analog computer

During the exploration of the different computational elements that make out the analog computer it was discovered that the operational amplifier plays a large role in analog computing. Based on the configuration of components around the operational amplifier a variety of different computational elements can be achieved. The operational amplifier is also a common and inexpensive component (under 1€ from most European vendors) that has seen a long time on the market. The IC versions of the components are also plentiful with different configurations of integrated operational amplifiers. The operational amplifier used in the artefact being created in this paper is the TL074 made by Texas Instruments [60] which features 4 operational amplifiers.

The accuracy of the analog computer is the sum of the components accuracy. Depending on the required accuracy of the analog computer the component accuracy should be selected accordingly.

The prototype analog computer uses polystyrene capacitors (Wima MKS2) that have high insulation, low leakage, low dielectric absorption and distortion [61]. These types of capacitor are widely available although more expensive that the common electrolytic capacitor (0.5€ for most values and can be had for as little as 0.1€ when bought in bulk). The accuracy of the capacitor used is 5%. This level of accuracy was chosen due to the wider availability and lower cost but still having a higher accuracy than the common 10%. The lower level of accuracy (compared to 1% or 0.1% capacitors) is a trade off to lower the over all cost.

The resistors used when constructing the prototype is metal film resistors with an accuracy of 1%. The metal film resistor was chosen due to it generating less noise compared to a carbon film resistor. This means that it is well adapted to high frequency applications which is beneficial for analog computation. The metal film resistor also features a long term stability (longer compared to carbon film resistors) and is less sensitive to temperature (meaning the resistor is more stable during temperature fluctuations) [62]. The coefficient potentiometers used are mono linear such as the RV9312NO model.

The modular analog computer that has been designed is a single board computer with different computational elements (integrators, inverters, multiplier and coefficient potentiometers). The analog computers different computational elements can be connected in series with each other. This is where the actual programming of the computer is done. The results of the calculations can be viewed by connecting a oscilloscope to the circuit. The computer and all the computational elements schematics can be found in the appendix A-D. A 3D rendition of the V1 of the computer can be seen in fig 28

E. *Integrator*

One of the core pillars of the analog computer is the integrator. As previously explained the integrator has several components such as capacitors, resistors and op amp. The integrator also has the ability to be introduced to an initial condition that initializes the computation.

The time factor can be set using the pin headers labeled integrator 1-4. Depending on how one sets the jumper clip, either a time factor of 1 or 10 can be achieved. The incoming resistor has a value of 100k and the two capacitors the user can choose between is 1nf and 10nf (capacitance value C). If the user chooses the 1nf cap a time factor of 1 is achieved, and if 10nf is chosen a factor of 10 if the resistor value is 100k (resistance value R). The user also has the opportunity to use an custom value capacitor so the time factor can be set to whatever the user wants. The mathematical expression of which is as mentioned before:

$$T = RC$$

The user can also introduce an IC (initial condition) to the integrators by the female header marked accordingly. The initial condition is triggered by pressing the switch. The switch will energize the two double pole relays that closes the connection for the initial condition. The integrators can be freely interconnected and connected to other modules via the 2x4p female header.

The computer has an on board voltage converter supplying the module with $-V$. The user has to supply the board with $+V$ and GND. The computer also has a 3p female header to supply other modules with the the same $\pm V$ and GND.

F. *Summer*

Similarly to the integrator module, the summer module is also based on a TL074CN chip. Based on the mathematical explanation described earlier the gain factor a_i can be expressed as:

$$a_i = \frac{R_f}{R_i}$$

The summers consists of a 100k resistors that act as the R_f. User can choose between two different values of R_i, 100k an 10k. Depending on what the user chooses the gain factor a_i will be either 1 or 10. The user can choose freely between the values. There two fist inlets are 100k and the third is 10k. The last inlet is empty to let the user specify their own value for R_i. The module has a 4p female header as output for the different summers and all the different summers can be freely connected with each other or other on board elements.

G. *Coefficient*

The analog computer inputs different coefficients by using potentiometers. The analog computer has six different potentiometers that can be used as coefficients for different calculations. The values of the potentiometers are 100k and have 2 open ports each (IN and OUT), all potentiometers have one leg connected to ground.

H. *Inverter*

The computer features features an inverter section. This sections enables the user to invert the voltage when necessary. The inverter is based on a TL074 op amp chip. The inverter consist of two 100k resistors configured as a summer with only one inlet. The inverting section has 4 inverters that can be connected freely to other computing elements.

I. *Multiplier*

The multiplier is a fairly simple and straight forward computational element to implement. Ulmann mentions in his book *"Analog and Hybrid Computer programming"* [5], that the analog multiplier can be built with multiple operational amplifiers. However, today there exists a range of ICs that do analog multiplication just as efficiently, cheaply and without the hassle of having to build your own multiplier [5]. The IC that offered the necessary specifications was the AD633ARZ with a 2% error margin. This multiplier is cheap (compared to the alternatives such as the MY634 which offer higher precision at 0.5% error margin at a significant price increase) analog multiplier that is widely available in different packages.

The IC AD633ARZ takes two inputs either inverted (X2, Y2) or non inverted (X1, Y1), multiplies the voltages and divides them by 10V before outputting the sum through the W pin. The last feature this chip has is a post multiplication summing junction in the form of the Z input. This input adds any voltage that is fed into it to the sum as an offset.

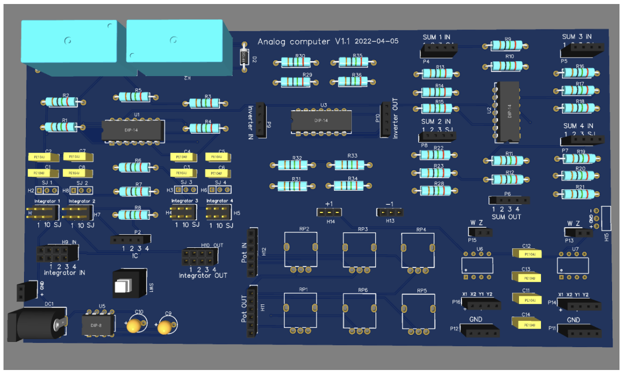


Fig. 28. A 3D rendition of V1

IX. Evaluation of the V1 Analog computer

A. *Integrator*

It was realized during assembly and testing that V1s integrator only had input through one resistor which meant that the computer only had one input for its integrator. This in turn limited the use of the integrator and was noted in the next iteration of the computer.

It was also decided to remove the onboard capacitors in favor of only an open female header. This meant that the user could use any desired capacitance value that can be inserted in the header to control the time factor.

In addition to this, it was decided to increase the number of integrators on the computer to effectively increase the number of equations that can be solved using the machine.

B. Inverter/Summer

While using the computer it was realized that the inverter and the summer basically did the same thing. If the summer was used by a factor of one with only one input, it was indistinguishable from an inverter. This is something that will be revised in V2 as it would seem unnecessary to have an inverter as it essentially is a less versatile summer.

C. Multiplayer

The multiplier has all its pins connected directly to a header despite only ever two of its input lines being used. This will be simplified in the upcoming version but limit the usage of inverse multiplication.

D. Potentiometers

The wiring of the potentiometers in V1 is correct with one of the ends going to ground and the wiper, and the other going to the input and output. However the layout of this version is a little confusing as the inputs and outputs are mismatched and unlabeled. This was addressed by making the layout of the board a lot clearer for the user.

E. Design choices

When using the analog computer V1 the fact that the headers were pretty spread out became obvious. Because of this, longer wires were required to program the computer which proved to be cumbersome to maneuver around the circuit board. The longer wires also contribute to a non pleasant time in using the potentiometers. The spread out nature of the headers made the cables prone to run across the turning shafts of the potentiometers.

In addition to this the computer had no defined probe outlets causing the user to have to move the oscilloscope probe between different header connections in order to get an output from the computer. This proved to be a less than ideal solution.

F. Usability and function testing

As it turns out, the testing and experimentation with V1 of the analog computer mostly came back with negative results. Starting off the simple oscillator seen in fig 29.

probing this setup in the output of $-sin(\tau)$ gave the following expected output on the oscilloscope screen (see fig 30)

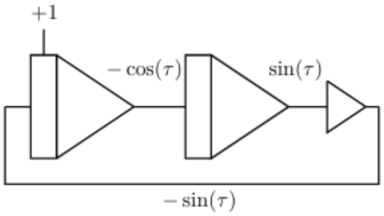


Fig. 29. simple oscillator [5]

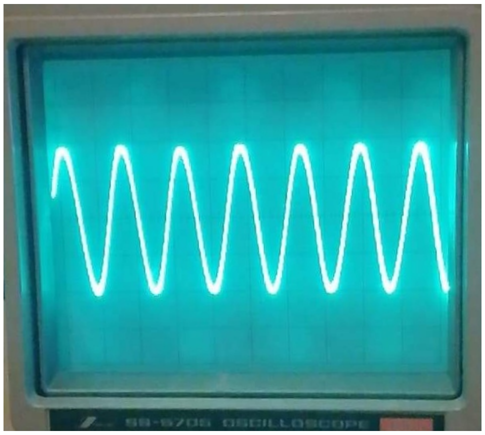


Fig. 30. simple oscillator output

The trial of the mass dampening was tested and the result did not follow the expected output of a damped spring. Instead the output came out like a capacitor charging and slowly discharging when the initial condition was introduced through the relays which indicated a fault within the routing of the integrator circuit.

Tests conducted with the Mathieu's equation yielded a similar output as the spring mass damping, a capacitor rapidly charging and slowly discharging. This was noted and reworked in the second revision.

G. Functionality evaluation

The first version of the analog computers integrator showed some inconsistent behavior. The introduction of the initial condition (IC) was not working as one would expect. The circuit could not properly oscillate and be triggered by the initial condition, thus proving that either the method of using relays or the initial condition was introduced to the circuit in a way that was not correct. The relays used in this version of the analog computer were electromagnetic double pole 12v relays (SMI-12VDC-SL-2C). The closing and opening times of the relays are less or equal to 10ms according to the manufacturer datasheet. This is problematic due to the computational time for the integrator is 1 - 10ms or less depending on the time factor. This means that the relay method is not suitable for introducing the initial condition as it introduced highly unreliable results.

The capacitors that allowed the user to use different time factors were also not ideal. If the user wanted to have separate time factors for two inputs (two separate inputs) it was not possible using the current computer. The integrators only had one inbound resistor (R1 in fig 17) thus limiting the inputs to the integrator as well as limiting the time scale factors to only one. To exemplify one could look at the integrator symbol in fig 18. Each one of the different inputs (e_i) has the potential to run on different time factors determined by the resistor and capacitor value as previously explained. To accommodate the usage of different time scale factors, one could use different inbound resistors. A possible solution could be to use a 10^8 and a 10^9 ohm resistor (100k and 1M) with a fixed value resistor such as 1nf. This would offer the user a choice in weight scale for the different inputs e_i

The summers, inverters, multiplier, and coefficient potentiometer present in the analog computer were working as expected. One problematic aspect present during the experiment was the fact that programming the analog computer was a bit cumbersome given the fact that all the in and outputs of the different computational elements were located far away from each other. This meant that there was no way to easily run 15cm DuPont-style wires between the different headers. The number of connection points was also too few in some instances. This was especially problematic when trying to create an output for the oscilloscope.

During experimentation, high levels of current were detected during both the introduction of IC to the circuit and when manipulating the coefficient potentiometers. The reasons for this were both the choice of using relays and using a lower value resistor such as the 100k as the "base level" resistor. The relays drew upwards of 200 mA upon activation at 12V, thus putting unnecessary additional strain on the board and wires making components and leads overheat.

The solution to these problems is to move the connection points closer together as well as having more connection points for the different computational elements. The resistors used in the circuit will also be looked over to both accommodate different time scale inputs to the integrators, as well as limit the current. The relays also need to be replaced with something else that does not draw as much current and has a faster or more controllable operating time.

X. EVALUATION OF THE V2 ANALOG COMPUTER

The complete schematic for the V2 analog computer can be found in appendix F-I. For the BOM file depicting components used, amount and price, see appendix J. Fig 31 depicts a 3D rendition of the V2 analog computer.

A. *Integrator*

The integrator of V2 was drastically redesigned when comparing it to the V1. The integrator featured on the V2 is not only more compact but also allows for more than one input at a time, which V1 was lacking. This was a severely limiting factor and drastically reduced the capabilities of the computer. The input number of the integrator is 8 where 3 are passed through 1Mohm resistors, 3 are passed through 100Kohm resistors, one is a summing junction (SJ) and one is designated to the initial condition (IC). V1's way of controlling the integration weight factor through the capacitor value was changed to be the resistor value as this allows the user to have more differing inputs into the integrator. The same idea of having an open capacitor position in the circuit was kept. The reason for this is that the machine time can be completely determined by the user but still allowing for a weighted input to the machine.

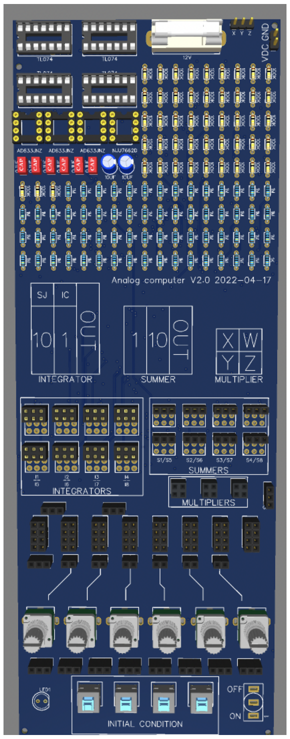


Fig. 31. A 3D rendition of the V2

As the trial with relays in V1 was a failure it was decided to go back to the basics that had proven to work on the breadboard. The solution was to use buttons for the IC switch. This worked very consistently when operating the machine but can be seen as a limiting factor when doing faster computations. As mentioned previously in this paper, MOS switches with contact bounce reducing circuits are ideal but hard to implement in a machine like this.

B. *Inverter/Summer*

As stated earlier the use of separate elements for the inventors and summers felt redundant as an inverter is a summer with a factor of one with only one input, which makes it simpler to only use summers. As with the inverters the input and output were made more compact while adding a greater number of input resistors. The new summer has 6 inputs 3 that are passed through 1Mohm resistors and 3 that are passed through 100Kohm resistors to give the differing factor values of one and ten.

C. *Multiplayer*

The multiplier has been simplified greatly in this version. instead of having all the inputs and outputs available X and Y two are tied to ground which allows the user to use the multiplier as a non inverting multiplayer. The voltage offset

(Z) and the output are both available as with the last iteration and just like the past elements it was compressed.

D. Potentiometers

The potentiometers are a clarified this time around as what input/output goes to what potentiometer. One negative aspect was the lack of an indicator of which is the input and which is the output. This is not a major issue but would be a good addition to future versions as connecting this element the wrong way can lead to the destruction of a potentiometer.

E. Design choices

With a more condensed control area this iteration of the analog computer was much easier to operate compared to the first one. Not having to maneuver around wires while engaging the potentiometers was a huge step forward in regards to the usability of the machine. Another added benefit is that the user no longer needs to search for longer wires when programming to further increase the size of the birds nest that was being created while programming.

The instructions of the computing elements printed on the board was also a good edition as a bigger set of instructions saved on having to look for tiny text next to the input/output headers (see fig 32).

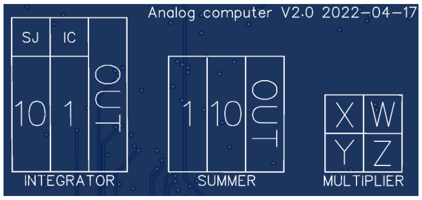


Fig. 32. Instruction print

One aspect that was noticed and improved upon was the xyz output terminal witch saves the user from having to disconnect and reconnect bulky oscilloscope probes when reprogramming or re-probing the computer (see fig 33).

Fig. 33. XYZ header terminal

In addition to all of this a number of headers connected by columns were added which turned out to be a great way to setup diodes for programs requiring them (see fig 34)

Fig. 34. open header columns

The analog computer preformed flawlessly when experimenting with both Mathieu's differential equation, simple oscillator and Spring mass dampening system. See fig 35, 36 and 37 for the results.

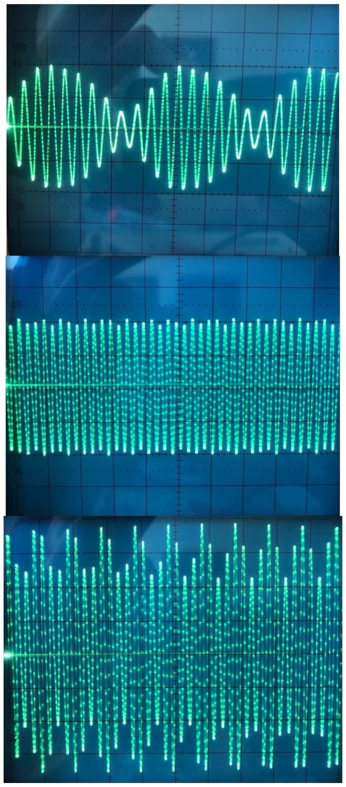


Fig. 35. Oscilloscope readout of Mathieu's equation from the V2 computer

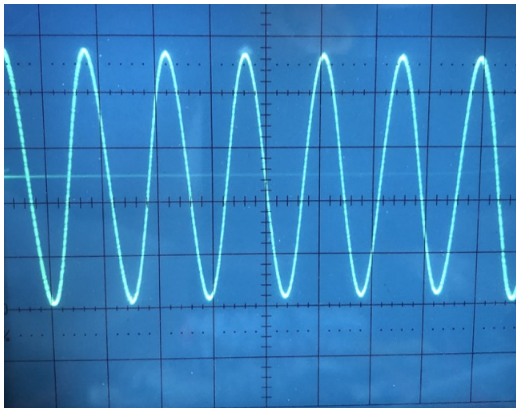


Fig. 36. Oscilloscope readout of simple oscillator the V2 computer

F. Hybrid system

In order to use the information produced by the analog computer in a hybrid set up, software for the Arduino UNO as well as a probe circuit was created. The Arduino UNO only accepts values from 0-5V, therefore the signal sent by the analog computer had to be split up and divided so as not to exceed this limit (see fig 38).

When connected in the configuration shown in fig 38 the Arduino UNO can be programmed with the following code to be used as an oscilloscope.

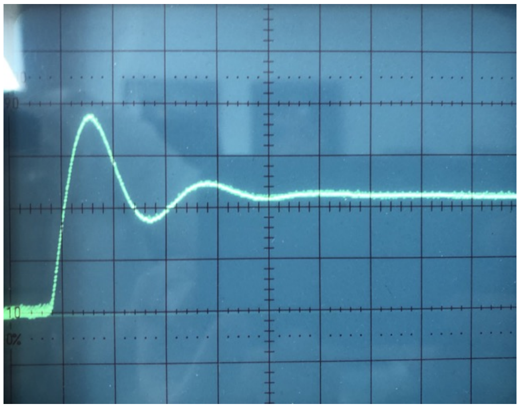


Fig. 37. Oscilloscope readout of Spring mass dampening the V2 computer

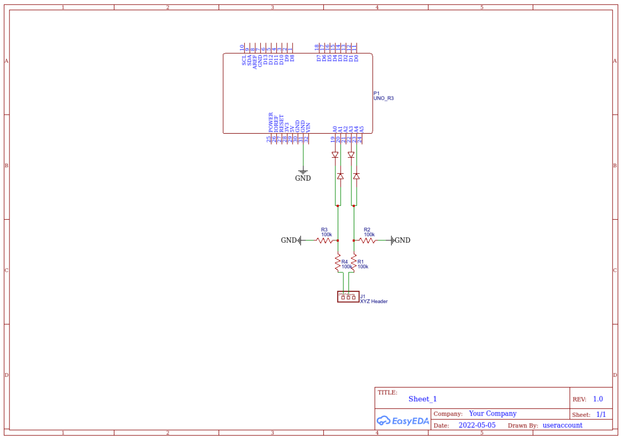


Fig. 38. Schematic of arduino probe

```
void setup() {
    Serial.begin(115200);
}

void loop() {

    // the Y probe
    int sensorValue0 = analogRead(A0);
    int sensorValue1 = analogRead(A1);

    // the X probe
    //int sensorValue3 = analogRead(A3);
    //int sensorValue4 = analogRead(A4);

    Serial.println(-sensorValue0);
    Serial.println(sensorValue1);
    //Serial.println(sensorValue3);
    //Serial.println(sensorValue4);

}
```

In fig 39 a demonstration of the Mathieu equation can be seen on a computer effectively using the Arduino UNO as an ADC. Important to note is that one might have to use another baud rate for different calculations.

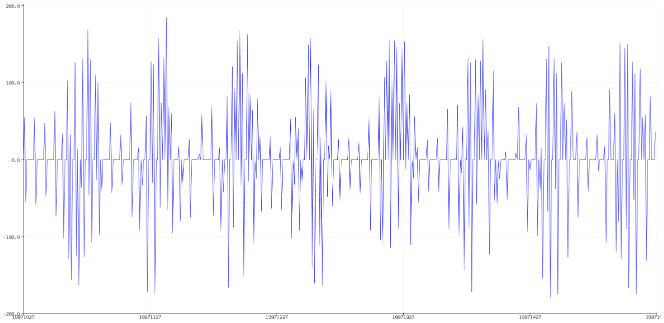
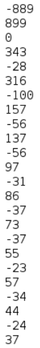


Fig. 39. Mathieu equation ploted output on a digital computer

If one wishes to only have the solutions for the equations as a numerical representation one would have to modify the software a bit by putting the serial in monitor mode instead of plot as shown in fig 40 for spring mass dampening.



```
-889
899
0
343
-28
316
-100
157
-56
137
-56
97
-31
86
-37
73
-37
55
-23
57
-34
44
-24
37
```

Fig. 40. Spring mass dampening numerical output on a digital computer

XI. EVALUATION

A. Feasibility of creating a low cost modular single board analog computer

Based on the results presented in this paper it is possible to create a single board analog computer that can solve a variety of differential equations. The V2 of the analog computer presented in this paper is a feature-rich, low-cost, easy-to-use machine that can be used to solve a wide array of problems using analog technology.

Noteworthy is that the device is not an extreme precision device as it is approximated to have an error margin of around 5% when using unknown values of 5% capacitors. If the value is known, or more precise capacitors are used, the multiplier dictates the error margin of 2% due to its accuracy deviance of 2%.

The analog computer V2 offers a somewhat forgotten technology in a form factor that is accessible to a larger group of people than previous computers. The price point of recreating this project is also accessible for individuals and institutions. All the prices for the components are available in the BOM file (See appendix J).

B. Capabilities of the modular analog computer

1) Physical capabilities: When looking at the low-level physical capabilities of what the analog computer can do the computational elements that are present dictates the complexity of the problem that can be solved. The analog computer has 8 integrators, 8 summers, 3 multipliers, and 6 coefficient potentiometers that can all be used in programming. This is the core limiting factor in how complex the solutions can be to the differential equations. Depending on how one simplifies the problem that will be simulated, solutions to fairly complex problems can be solved. As proven by solving both Mathieu's differential equation and the spring-mass dampening equation the computer works flawlessly in simpler and straightforward problem-solving. One could argue that having more coefficient potentiometers could have offered a wider range of possibilities due to the larger amount of both integrators and summers. On the other hand, the addition of three multipliers generates the opportunity to solve more complex problems. An example that has been briefly mentioned in this paper, but should be acknowledged due to it being widespread, is the Lorenz attractor.

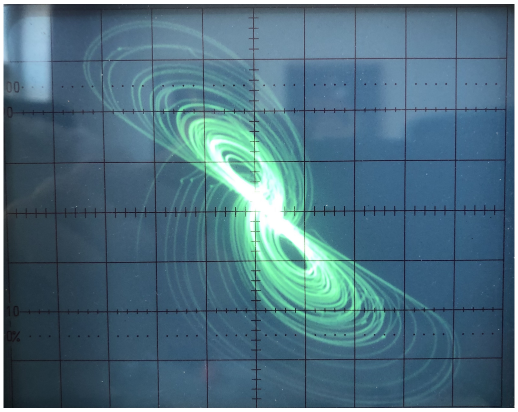


Fig. 41. Lorentz attractor from the computer

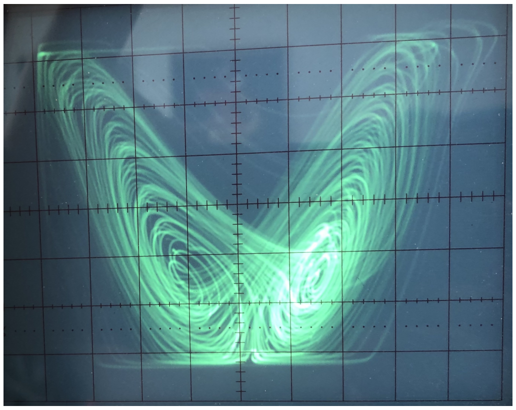


Fig. 42. Lorentz attractor from another angle, from the computer

What this means is that the physical limitations of the analog computer stretches outside the scope of the experiments to prove that it works. The applications of the artifact are therefore highly increased to encompass both the differential equations used as baseline functionality tests to simpler chaotic systems.

One limiting factor to the usability of an analog computer is the accuracy of the analog representation of the physical system being studied. If one assumes that the equation has been translated correctly to represent the actual phenomena, the limiting factor is the accuracy of the individual components. The computers limiting components for accuracy are both the capacitors and the multiplier. The capacitors user has an accuracy of 5% and the multiplier at 2%. Since the computer has not been designed with high accuracy as the main focus, one could assume the accuracy to be closer to 5% given the choice of components. This could be remedied by using more precise capacitors which would not be something complicated for the operator. The current setup uses open female headers to accommodate different values of capacitors. This will therefore give the user a wide array of machine time factors to operate under. The capacitors can therefore be swapped out with ones that offer higher precision and thus result in a more precise machine.

2) Capabilities for providing academic value: If one looks at the wider capabilities of the analog computer, the value it can bring as an academic tool can not be disregarded. As previously mentioned, most students seldom actually solve differential equations. The students merely create expressions that satisfy the equation. The process of which is often only done in a predetermined and mechanical way based on a set of rules. This is not ideal for acquiring deeper knowledge about the process and solution. This problem relates both to maths and physics classes. One direct possible application could be to apply the analog computer in control engineering classes. The analog computer created has proven to have the capability to solve differential equations that would have otherwise only been solved using analytical solutions. Programming the analog computer requires knowledge about electronics and their role in calculations. Therefore the analog computer could bring more value than just being a way to plot a differential equation. The Analog computer would be a cost-effective way to increase students' knowledge about math, physics, and electronics to further the understanding and connection between them.

A low-cost analog computer is something that has been a topic for academia previously. The main problem is the somewhat dated form factor and functionality. The low-cost analog computer theorized previously has aimed to mimic the larger more capable machines. The main problem with this is both the larger form factors and physical shape that limit the effective use of space. Even the smaller desk-sized units presented in this paper (Except for THAT) are so large they become cumbersome to include in teaching and be used by enthusiasts. The artifact that has been created in this paper (the analog computer) brings the form factor of a single board computer to the realm of analog computing. This includes programming the computer using DuPont-style connectors instead of the more conventional banana plug. This liberates analog computing from a high upfront cost and saves space, but also limits the complexity of the programming possible due to not being able to stack the connectors (like stackable banana plug cable) being used. This was addressed by having more possible connecting points for both the summers and integrators, thus limiting the negative impact.

Most academic institutions have discarded analog technology as something ancient and outdated. This has led to the knowledge of analog computing being lost to time with a few exceptions mentioned in this paper. Most digital systems that mimic analog representations are an approximation at best that requires a lot of energy to depict an abstraction of the real world. The main problem with implementing analog technology in a digital world is the fact that few people know about the strengths of analog computing. Even fewer people know what an analog computer is and how to use it. To combat this fact, an easy way to re-introduce analog computing into society is needed. The artifact created in this paper can be a part of the much larger re-institution of analog computing in a digital society. This ties in with the extended capability of the artifact created.

3) The extended capability of the analog computer: The success of the Arduino platform can partly be attributed to its connection to the much larger platform and community. The analog computer THAT was created by Anabrid has the same characteristics in regards to its connection to a larger platform and community. The collectives' ability to foster and continue the elaboration and maturing of technology has proven to be a powerful tool. The extended capability of the artifact created is therefore a part of a much larger community. The artifact has the potential due to its low cost, simple construction, and high usability to expand the knowledge of analog computing to a wider group of people than before. A parallel to both THAT and the artifact created in this paper can be the creation of Things such as Bjögvinsson et al [38] describes. The artifact in this paper could play a much larger role in socio-material assemblies. The definition pushes for the artifact to generate controversies, opening up new ways of thinking and behaving as well as being ready for unexpected use. Analog technology in general is a lost art due to it being left behind due to the fast development of the digital computer. This means that applying analog technology to solve problems in a digital era could be considered an avant-garde approach to solving contemporary problems. As proven in literature, the analog computer can be applied to modern problems (like ML algorithms [22]) and excel in both computational power, speed, and energy efficiency. This forces the individual to open up to new ways of thinking and behaving as well as the somewhat "ancient" analog technology to become ready for unexpected use in a digital society.

4) General applications of the analog computer in a digital society: This paper argues that the analog computer has a role in a digital society. The ability to solve complex problems (differential equations) quickly and efficiently is something the analog computer can provide. If the application to a specific problem does not require extreme precision and can be discretized by approximating the differentials so the computer can integrate in machine time, an analog computer can be used.

Classic applications of analog technology are within the field of control engineering. This includes nuclear power plant control units, heat distribution, and automotive suspension/-traction control. The core advantage of using analog technology is energy efficiency and computational speed. Important to note is the example given by Köppel et al. [2]. It was proven that given an increasing problem size the analog computers' time to solution was constant while the digital computers' time to solution was increasing with the problem size. This means that the time to solution for analog computers is not affected by the problem size while the digital is. The energy consumption for the increasing problem size was for analog computers increasing with the problem size and for digital constant. This means that the analog computers' power consumption is dependent on the problem size. This is due to the number of components that increases when programming complex problems. This means that the analog computer only uses as much power as it needed to solve a specific problem.

Concrete examples of applications could be to calculate bed heating and extrusion heating for 3D-printers, thus effectively controlling the heating using analog technology. Even IoT devices such as drones could benefit from analog technology. Even with the introduction of 5G, the speed at which information can be sent is not enough in some applications. In order for the drone to stabilize it has to know what forces are applied to it. This requires a lot of processing that could have been done on the server level but due to the information traveling too slow is not possible. Analog technology could have been implemented in a use case similar to this to have more energy-efficient devices since analog technology uses orders of magnitude less power. The artifact created could be used in research similar to this. It could also be used in control engineering problems such as balancing a ball inside a magnetic field where computational speed is key. The artifact created in this paper could theoretically be used in an application such as magnetic field suspension as described by Huang [53], mentioned earlier in this paper, due to its very capable and high-speed computation abilities. The high-speed capabilities of analog technology could especially be implemented in systems like the ESS in Lund. The speed at which calculations are needed to run at is something that digital computers are not well equipped for, but analog computers can do easily. An analog/hybrid system could potentially save both energy and money and increase speed in special cases like this.

A novel and widespread application for analog computing are within the field of AI and neural networks. Recent findings in neuroscience have proven that digital computers are not strictly an analog for how the human brain works. If one should take these findings into account for the development of neural networks, analog technology could be implemented to get a better analog for the human brain, thus improving neural networks. There are also several solutions on an analog chip technology to better solve specific calculations concerning AI. These analog solutions include the aforementioned crossbar arrays, ferroelectric devices, electrochemical devices, and restive Random-Access Memory. These emerging in-process memory solutions are all to mitigate the Newman bottlenecks that plague digital computation today. A good testimony to the potential of analog chip technology is the Very-Large-Scale-Integrated Analog Computer (VLSI) which can rival most Green500-list digital alternatives when solving differential equations while delivering 21 gigaflops per watt. If one were to implement this in server and/or data centers, the potential energy savings would be substantial and result in a positive impact on a now extremely energy-hungry industry.

The artifact created in this paper is a feature-full device that can be used as both a prototyping aid when investigating the usage and impact of analog technology on a specific problem and as an educational tool. The external equipment needed is fairly simple (regular PSU, oscilloscope, and wires for programming) and can therefore be used to quickly set up and integrated into a wide array of applications. The computer can also be used as an analog co-processor by utilizing an Arduino UNO and its ADC chip. The additional feature of a non-specified machine time can also be heavily utilized by the operator to set a specific integral computation time to solve specific problems or simulate problems using different time variables.

XII. CONCLUSION & FUTURE WORK

This project has proven that an analog/hybrid computer with the capabilities to calculate second order differential equations can be produced on a small budget. While not being the most accurate of analog computing devices the artifact produced in this project shows great opportunities both now and for future iterations.

The need for energy efficient-high power computing devices has never been greater and will only continue to grow as time goes on. If the world wants to see any real change in performance as well as energy efficiency it needs to embrace analog computing. One of the greatest places to start technological changes is in the educational institutions as it is the birthplace of both learning and knowledge.

The view of analog computing emerging during this project is that it is a highly useful and powerful tool that nobody even knows. Analog computation is still relevant even though it was forgotten due to the breakthrough of digital computation. Hence the creation of a learning aid that cheaply allows a student to apply the method “learning by doing”, a method very often used when learning to write code or similar in the digital domain. An analog device such as the artifact in this paper is of great value to the future development of our society. Like Ulmann mentions in his article *"Why algorithms suck and analog computers are the future"*:

“Clearly analog computing holds great promise for the future. One of the main problems to tackle will be that programming analog computers differ completely from everything students learn in university today.” [1]

What universities need to conduct effective education when it comes to programming are efficient, cheap, and easy-to-use computers that the students have access to. This holds true for both digital and analog computing. This is where the project in this paper adds to the scientific community by proposing a cheap and easy-to-use introduction to analog computers.

With this being said, future work needs to be conducted within educational establishments. Finding people who are interested and willing to conduct education within this reawakening field is a necessity. Implementing and using tools like the one proposed in this paper is crucial to the future development of computer science and a more sustainable future.

Future work also includes the analog computer proposed in this paper. There is still more work that can be done in order to improve on the current iteration of the artifact. Future iterations could for instance have an even smaller form factor, some sort of indicator for the potentiometers, and perhaps even be controlled entirely from a micro-controller of some sort as a hybrid controller. This remains to be explored in future studies.

REFERENCES

[1] B. Ulmann, "Why algorithms suck and analog computers are the future," 2017. [Online]. Available: https://blog.degruyter.com/algorithms-suck-analog-computers-future/

[2] S. Köppel, B. Ulmann, L. Heimann, and D. Killat, "Using analog computers in today's largest computational challenges," *Advances in Radio Science*, vol. 19, no. D., pp. 105–116, 2021.

[3] B. Ulmann, *Analog computing*. Oldenbourg Wissenschaftsverlag, 2013, pp. 1–55, 170–190.

[4] "Anabrid," https://www.anabrid.com/index.html, accessed: 2022-02-02.

[5] B. Ulmann, *Analog and Hybrid Computer Programming*. Walter de Gruyter, 2020, pp. 1–55, 109–120.

[6] computerhistory.org, "Bush's analog solution: The differential analyzer," 2022. [Online]. Available: https://www.computerhistory.org/revolution/analog-computers/3/143

[7] W. Thomson, "V. mechanical integration of the linear differential equations of the second order with variable coefficients," *Proceedings of the Royal Society of London*, vol. 24, no. 164-170, pp. 269–271, 1876.

[8] Y. Tsividis, "Not your father's analog computer," *IEEE Spectrum*, vol. 55, no. 2, pp. 38–43, 2018.

[9] W. Cameron and G. Taylor, "Examples of analog computer applications in nuclear processes," *Simulation*, vol. 2, no. 4, pp. 23–38, 1964.

[10] H. Talmadge Jr, "An orbit computer for a satellite position display," NAVAL RESEARCH LAB WASHINGTON DC, Tech. Rep., 1961.

[11] R. H. Good and O. Piccioni, "Analog computer for charged particle trajectories," *Review of Scientific Instruments*, vol. 31, no. 10, pp. 1035–1039, 1960.

[12] technikum29, "Analog and hybrid computers," 2022. [Online]. Available: https://www.technikum29.de/en/computer/analog.php

[13] "Anabrid - the analog thing," https://the-analog-thing.org/, accessed: 2022-02-02.

[14] K. Steiglitz, *8. Analog Computers*. Princeton University Press, 2019, pp. 107–131. [Online]. Available: https://doi.org/10.1515/9780691184173-009

[15] R. Hintemann and S. Hinterholzer, "Energy consumption of data centers worldwide," *Business, Computer Science (ICT4S)*, 2019.

[16] M. Dayarathna, Y. Wen, and R. Fan, "Data center energy consumption modeling: A survey," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 1, pp. 732–794, 2015.

[17] K. Peffers, T. Tuunanen, and B. Niehaves, "Design science research genres: introduction to the special issue on exemplars and criteria for applicable design science research," *European Journal of Information Systems*, vol. 27, no. 2, pp. 129–139, 2018. [Online]. Available: https://doi.org/10.1080/0960085X.2018.1458066

[18] S. T. March and G. F. Smith, "Design and natural science research on information technology," *Decision Support Systems*, vol. 15, no. 4, pp. 251–266, 1995. [Online]. Available: https://www.sciencedirect.com/science/article/pii/0167923694000412

[19] J. F. Nunamaker, "A methodology for the design and optimization of information processing systems," in *Proceedings of the May 18-20, 1971, Spring Joint Computer Conference*, ser. AFIPS '71 (Spring). New York, NY, USA: Association for Computing Machinery, 1971, p. 283–294. [Online]. Available: https://doi.org/10.1145/1478786.1478826

[20] J. G. Walls, G. R. Widmeyer, and O. A. El Sawy, "Building an information system design theory for vigilant eis," *Information Systems Research*, vol. 3, no. 1, pp. 36–59, 1992. [Online]. Available: https://doi.org/10.1287/isre.3.1.36

[21] K. Peffers, T. Tuunanen, M. Rothenberger, and S. Chatterjee, "A design science research methodology for information systems research," *Journal of Management Information Systems*, vol. 24, pp. 45–77, 01 2007.

[22] P. A. Holst, "Analog computer," in *Encyclopedia of Computer Science*, 2003, pp. 53–59.

[23] K. Eriksson and H. Gavel, *Diskret matematik och diskreta modeller*. Studentlitteratur, 2013.

[24] W. K. Grassmann and J.-P. Tremblay, "Logic and discrete mathematics," *Upper Saddle River, NM: Prentice Hall*, 1996.

[25] B. Ulmann, "Lecture: Analog computer returns," May 2019. [Online]. Available: https://www.youtube.com/watch?v=sVKmiCy4LA8

[26] M. Braun and M. Golubitsky, *Differential equations and their applications*. Springer, 1983, vol. 1.

[27] D. B. Ulmann. (2017). [Online]. Available: https://analogparadigm.com/downloads/alpaca_10.pdf

[28] M. Holzer and B. Ulmann, "Hybrid computer approach to train a machine learning system," in *Handbook of Unconventional Computing: VOLUME 2: Implementations*. World Scientific, 2022, pp. 303–341.

[29] D. W. Hagelbarger, C. E. Howe, and R. M. Howe, "Investigation of the utility of an electronic analog computer in engineering problems," 1949.

[30] L. D. Kov Ach and W. Comley, "The analog computer as a teaching aid in diffential equations," *Mathematics and Computers in Simulation*, vol. 3, no. 2, pp. 60–63, 1961. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0378475461800244

[31] B. Rasof and R. Tomovic, "The analog computer as an aid in teaching mathematics," *Mathematics and Computers in Simulation*, vol. 4, no. 4, pp. 179–183, 1962. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S037847546280024X

[32] W. S. Adams and C. Volz, "The analog computer laboratory in engineering education," *IEEE Transactions on Education*, vol. E-7, no. 2/3, pp. 59–62, 1964.

[33] J. A. Hamilton, "An analog computer for educational laboratories," *Journal of Chemical Education*, vol. 52, no. 5, p. 340, 1975. [Online]. Available: https://doi.org/10.1021/ed052p340

[34] R. H. Spiess, "The comdyna gp-6 analog computer - twenty five years... and still counting," *SIMULATION*, vol. 59, no. 5, pp. 323–325, 1992. [Online]. Available: https://doi.org/10.1177/003754979205900510

[35] R. Spiess, "The comdyna gp-6 analog computer: alive but not exactly kicking," *IEEE Control Systems Magazine*, vol. 25, no. 3, pp. 68–73, 2005.

[36] "The analog paradigm," https://analogparadigm.com/index.html, accessed: 2022-02-15.

[37] "The analog thing," https://the-analog-thing.org/wiki/Main_Page, accessed: 2022-02-15.

[38] E. Bjögvinsson, P. Ehn, and P.-A. Hillgren, "Design things and design thinking: Contemporary participatory design challenges," *Design issues*, vol. 28, no. 3, pp. 101–116, 2012.

[39] D. J. Cuartielles Ruiz, "Platform design: Creating meaningful toolboxes when people meet," Ph.D. dissertation, Malmö University, Faculty of Culture and Society, 2018.

[40] C. J. Maley, "Toward analog neural computation," *Minds & machines*, no. 28, pp. 77–91, aug 2018. [Online]. Available: https://doi.org/10.1007/s11023-017-9442-5

[41] S. Ghosh-Dastidar and H. Adeli, "Spiking neural networks," *International journal of neural systems*, vol. 19, no. 04, pp. 295–308, 2009.

[42] S. G. Hormuzdi, M. A. Filippov, G. Mitropoulou, H. Monyer, and R. Bruzzone, "Electrical synapses: a dynamic signaling system that shapes the activity of neuronal networks," *Biochimica et Biophysica Acta (BBA)-Biomembranes*, vol. 1662, no. 1-2, pp. 113–137, 2004.

[43] G. Söhl, S. Maxeiner, and K. Willecke, "Expression and functions of neuronal gap junctions," *Nature reviews neuroscience*, vol. 6, no. 3, pp. 191–200, 2005.

[44] W. Haensch, T. Gokmen, and R. Puri, "The next generation of deep learning hardware: Analog computing," *Proceedings of the IEEE*, vol. 107, no. 1, pp. 108–122, 2019.

[45] G. W. Burr, A. Sebastian, T. Ando, and W. Haensch, "Ohm's law + kirchhoff's current law = better ai: Neural-network processing done in memory with analog circuits will save energy," *IEEE Spectrum*, vol. 58, no. 12, pp. 44–49, 2021.

[46] R. Eigenmann and D. J. Lilja, "Von neumann computers," *Wiley Encyclopedia of Electrical and Electronics Engineering*, vol. 23, pp. 387–400, 1998.

[47] U. Hasson, J. Chen, and C. J. Honey, "Hierarchical process memory: memory as an integral component of information processing," *Trends in cognitive sciences*, vol. 19, no. 6, pp. 304–313, 2015.

[48] G. Cowan, R. Melville, and Y. Tsividis, "A vlsi analog computer/digital computer accelerator," *IEEE Journal of Solid-State Circuits*, vol. 41, no. 1, pp. 42–53, 2006.

[49] J. M. Khurpade, D. Rao, and P. D. Sanghavi, "A survey on iot and 5g network," in *2018 International Conference on Smart City and Emerging Technology (ICSCET)*, 2018, pp. 1–3.

[50] M. Gomes, R. da Rosa Righi, and C. A. da Costa, "Internet of things scalability: Analyzing the bottlenecks and proposing alternatives," in *2014 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT)*, 2014, pp. 269–276.

[51] J. Dentler, S. Kannan, M. A. O. Mendez, and H. Voos, "A real-time model predictive position control with collision avoidance for commercial low-cost quadrotors," in *2016 IEEE Conference on Control Applications (CCA)*, 2016, pp. 519–525.

[52] O. Mitchell, “Self driving cars have power consumption problems,” 2018. [Online]. Available: https://www.therobotreport.com/self-driving-cars-power-consumption/

[53] H. Huang, H. Du, and W. Li, “Stability enhancement of magnetic levitation ball system with two controlled electomagnets,” in *2015 Australasian Universities Power Engineering Conference (AUPEC)*. IEEE, 2015, pp. 1–6.

[54] R. Visintini, M. Cautero, G. Göransson, C. Martins, and P. Torri, “Power converters for the ess warm magnets,” in *8th Int. Particle Accelerator Conf.(IPAC’17), Copenhagen, Denmark, 14â 19 May, 2017*. JACOW, Geneva, Switzerland, 2017, pp. 3372–3374.

[55] maximintegrated, “glossary definition for non-inverting op amp,” 2022. [Online]. Available: https://www.maximintegrated.com/en/glossary/definitions.mvp/term/Non-Inverting%20Op%20Amp/gpk/1219

[56] D. Normen, “Analogrechtentechnik din 40700,” 1969. [Online]. Available: https://www.rsp-italy.it/Electronics/Analog%20computing/_contents/DIN40700%201969.pdf

[57] R. Mancini, *Op amps for everyone: design reference*. Newnes, 2003.

[58] B. Carter and T. R. Brown, *Handbook of operational amplifier applications*. Texas Instruments Dallas, TX, 2001.

[59] A. Devices, “Ad633 analog multiplier,” 2018. [Online]. Available: https://www.analog.com/media/en/technical-documentation/data-sheets/AD633-EP.pdf

[60] T. Instruments, “Tl074cn-datasheet,” 2022. [Online]. Available: https://www.ti.com/general/docs/suppproductinfo.tsp?distId=26&gotoUrl=http%3A%2F%2Fwww.ti.com%2Flit%2Fgpn%2Ftl074

[61] L. A. Matheson and V. J. Caldecourt, “Electrical charge storage in polystyrene capacitors,” *Journal of Applied Physics*, vol. 22, no. 9, pp. 1176–1178, 1951. [Online]. Available: https://doi.org/10.1063/1.1700128

[62] G. Siddall and B. A. Probyn, “Vacuum-deposited metal film resistors,” *British Journal of Applied Physics*, vol. 12, no. 12, pp. 668–674, dec 1961. [Online]. Available: https://doi.org/10.1088/0508-3443/12/12/306

APPENDIX A
INTEGRATOR SCHEMATIC

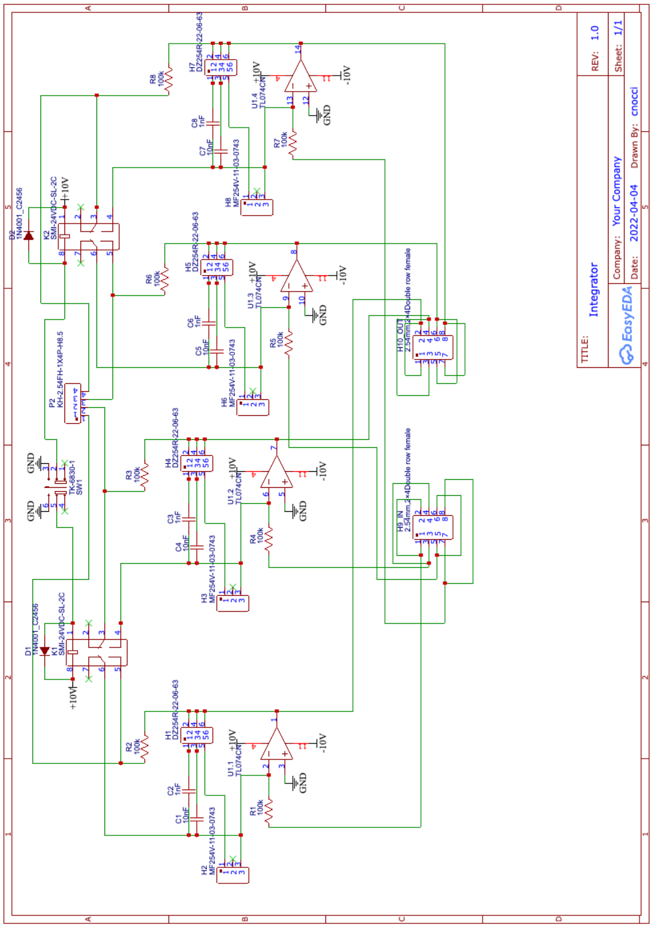


Fig. 43. Integrator schematic

APPENDIX B
POWER SUPPLY SCHEMATIC

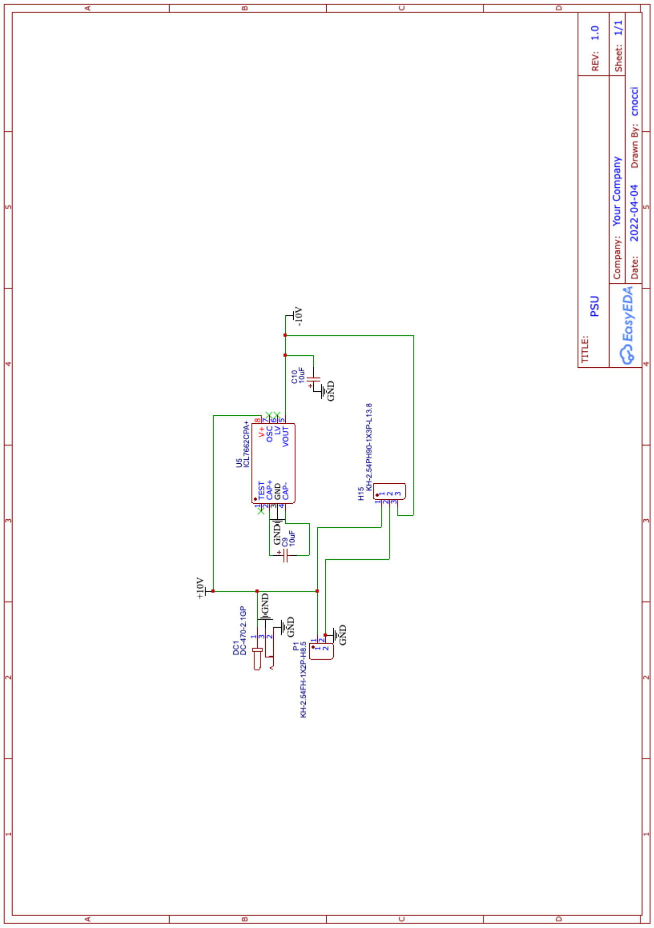


Fig. 44. Power supply schematic

APPENDIX C
SUMMER SCHEMATIC

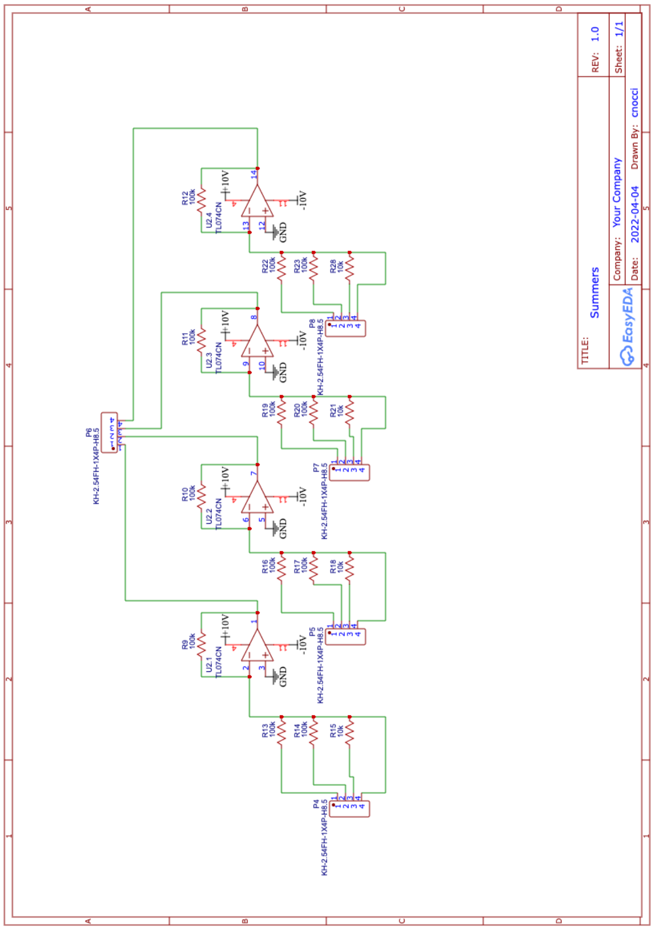


Fig. 45. Summer schematic

APPENDIX D
SUMMER SCHEMATIC

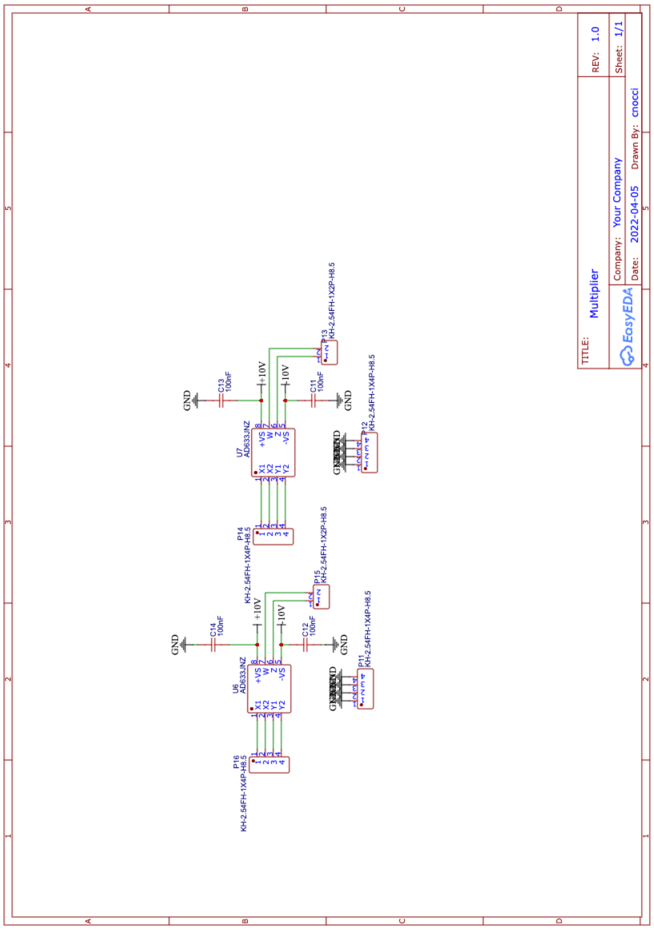


Fig. 46. Multiplier schematic

APPENDIX E
HAMILTON ANALOG COMPUTER SCHEMATIC

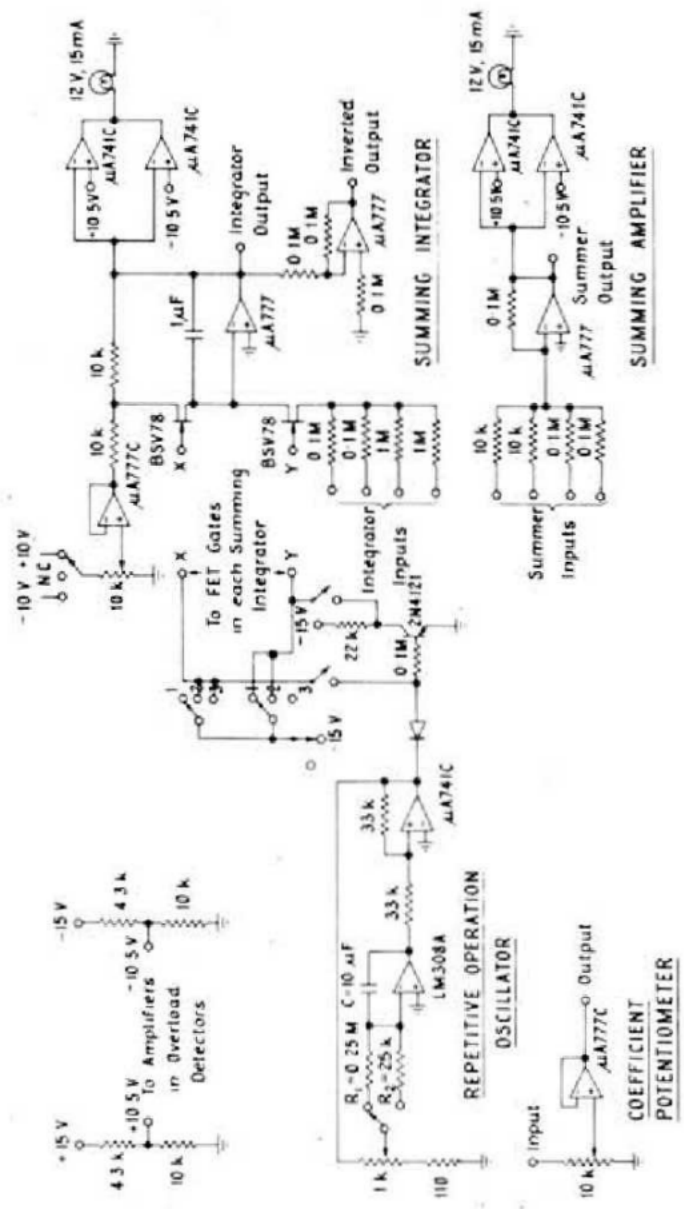


Fig. 47. The schematics for J. A. Hamilton analog computer [33]

APPENDIX F
INTEGRATOR V2 SCHEMATIC

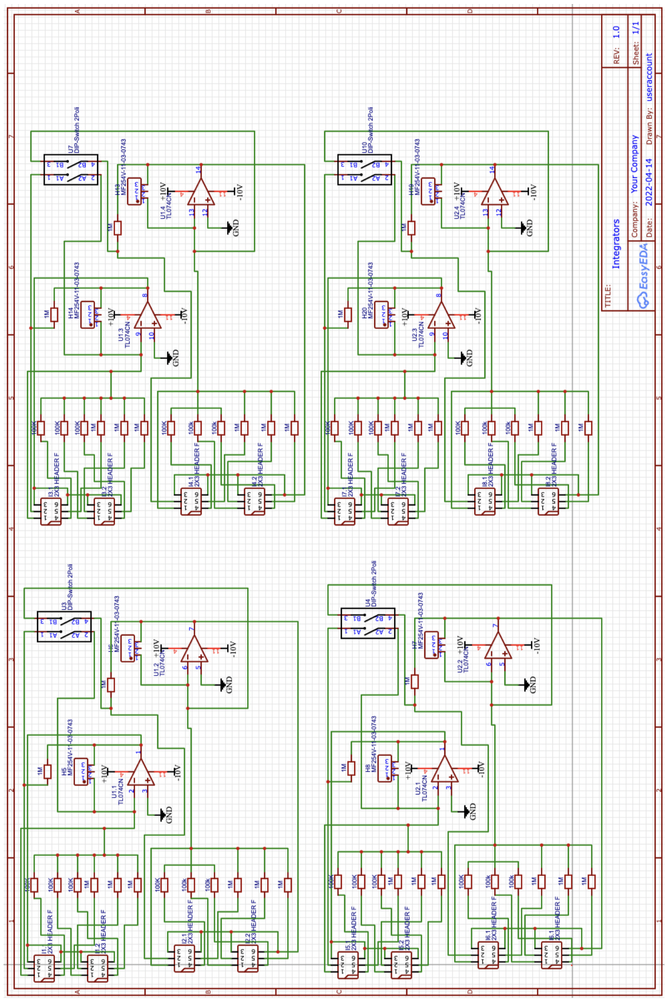


Fig. 48. integrator schematic analog computer V2

APPENDIX G
SUMMER V2 SCHEMATIC

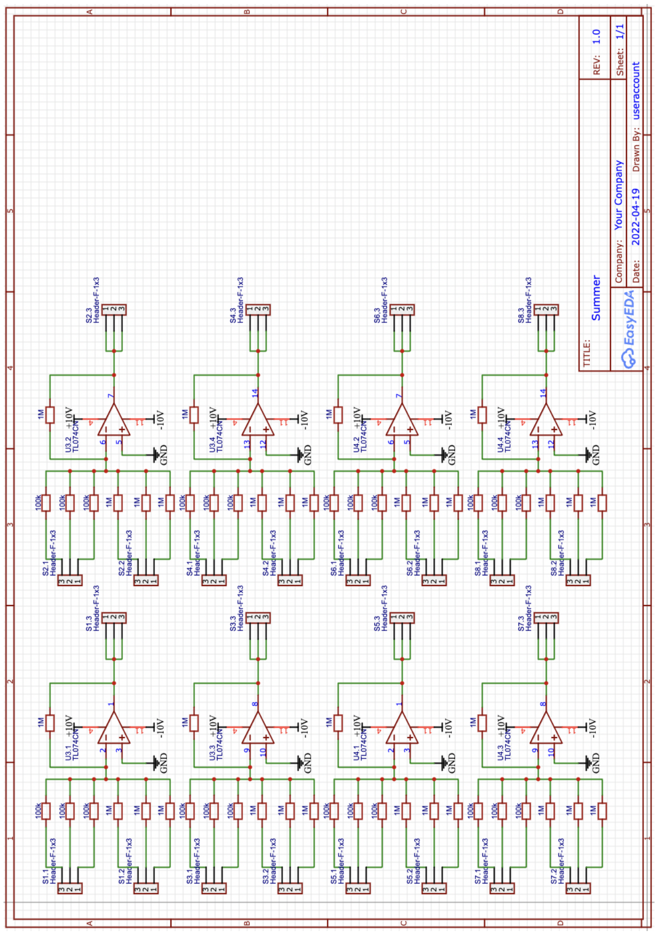


Fig. 49. Summer schematic analog computer V2

APPENDIX H
MULTIPLIER V2 SCHEMATIC

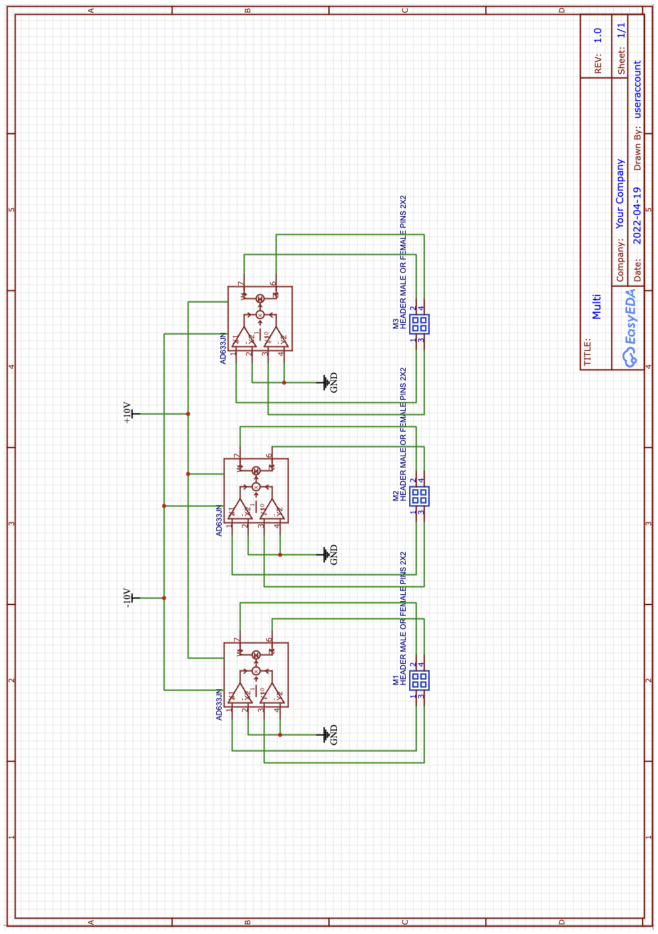


Fig. 50. Multiplier schematic analog computer V2

APPENDIX I
PSU AND POTENTIOMETER V2 SCHEMATIC

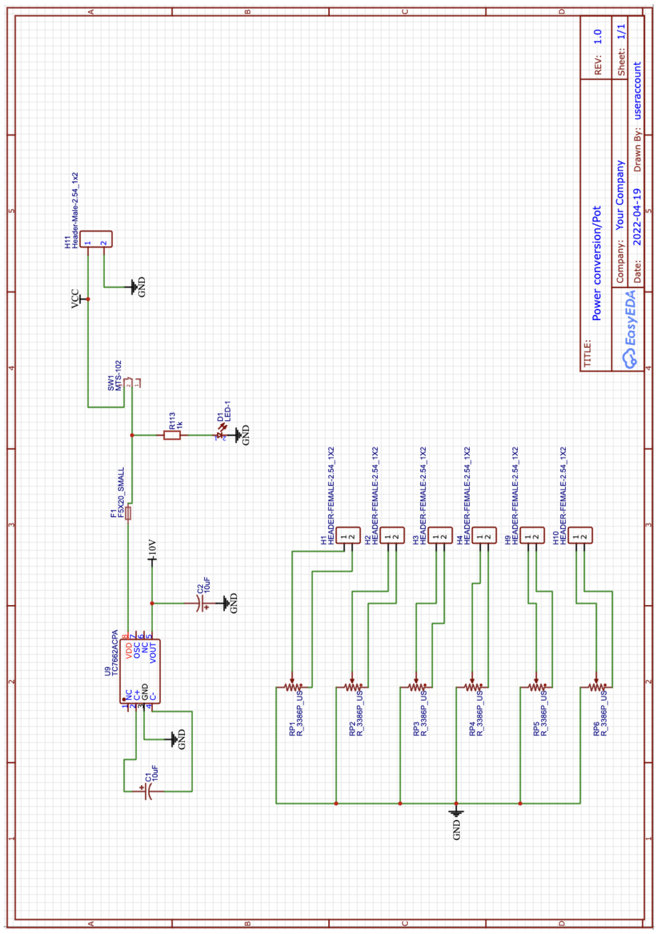


Fig. 51. PSU & Potentiometer schematic analog computer V2

APPENDIX J
BOM-TABLE OF V2

TABLE I
BOM-TABLE OF V2

ID	Name	Designator	Footprint	Quantity	Price for one when buying 100
1	MTS-102	SW1	MTS-102	1	0,409 €
2	LED_LED5MM	LED1	LED_LED5MM	1	0,494 €
3	BUTTON(PS850L)1	I1/I2,I3/I4,I5/I6,I7/I8	BUTTON(PS850L)1	4	1,10 €
4	RESISTOR-MRF-12	1M	RESISTOR-MRF-12	73	0.07 €
5	RESISTOR-MRF-12	1K	RESISTOR-MRF-12	1	0.07 €
6	RESISTOR-MRF-12	100K	RESISTOR-MRF-12	39	0.07 €
7	PINHEAD2X1WIDE	ST1	PINHEAD2X1WIDE	1	0.16 €
8	PINHEADER-3X1	GND,I1-CAP,I7-CAP,I4-CAP,I3-CAP,I2-CAP,I8-CAP,I6-CAP,I5-CAP,+1,-1,V3	PINHEADER-3X1	20	0.17 €
9	DIP-8_L10.0-W6.5-P2.54-LS7.6-BL	NJU7662D,AD633JNZ,AD633JNZ,AD633JNZ	DIP-8_L10.0-W6.5-P2.54-LS7.6-BL	3	12,68 €
10	DIP-8_L10.0-W6.5-P2.54-LS7.6-BL	NJU7662D	DIP-8_L10.0-W6.5-P2.54-LS7.6-BL	1	1,77 €
11	DIP-14_L17.8-W10.2-P2.54-BL	TL074,TL074,TL074,TL074	DIP-14_L17.8-W10.2-P2.54-BL	4	0,744 €
12	HDR-3X2/2.54	P1,S1/S5,P3,P4,P5,P6,P7,P8	HDR-3X2/2.54	8	0.25 €
13	HEADER-2X2	M1,P10,P11	HEADER-2X2	3	0.96 €
14	HEADER_2X4-2.54P	U9,I1,U11,U12,U13,U14,U15	HEADER_2X4-2.54P	16	0.53 €
15	HEADER-2X4	H9,H10,H11,H12,H13,H14,H15,H16	HEADER-2X4	8	0.53 €
16	ALPHA 9MM	VR1,VR2,VR3,VR4,VR5,VR6	ALPHA 9MM	6	0.17 €
17	HEADER-FEMALE-2X1-2.54	J1,J2,J3,J4,J5,J6	HEADER-FEMALE-2X1-2.54	6	0.16 €
18	HEADER MALE 3X1	U18	HEADER MALE 3X1	1	0.17 €
19	FUSE_FUSE_HOLDER_5X20MM	12V	FUSE_FUSE_HOLDER_5X20MM	1	0.55 €
20	PKG-ELE-CAP-D5.0XH11.0XF2.0-VERT	10UF,10UF	PKG-ELE-CAP-D5.0XH11.0XF2.0-VERT	2	0.38 €
21	RAD-0.1	0.1UF,0.1UF,0.1UF,0.1UF,0.1UF,0.1UF	RAD-0.1	6	0.17 €