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*In This Issue*

## How To Calculate Fin Flutter Speed



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# How To Calculate Fin Flutter Speed

By Zachary Howard

After construction completed on July 1, 1940, the Tacoma Narrows Bridge was the third largest suspension bridge in the entire world, behind the Golden Gate Bridge and the George Washington Bridge. Its infamy lies not with historic length but in its nickname, Galloping Gertie. The nickname arose from the bridge's easily excitable bending mode. Drivers would watch the oncoming cars rise and fall with the violent motion of the bridge. During a particularly strong forty-mile per hour gust the newly excited torsion mode of the bridge caused a violent twisting along the centerline of the bridge. Figure 1 below shows the bending and torsional modes of the bridge. Despite being made from carbon steel and concrete on, November 7, 1940 the growing torsional oscillations overwhelmed the natural damping of the bridge and Gertie plunged 300 ft into the ocean below. After months of research NACA engineers diagnosed the cause of the vibrations as aeroelastic flutter.

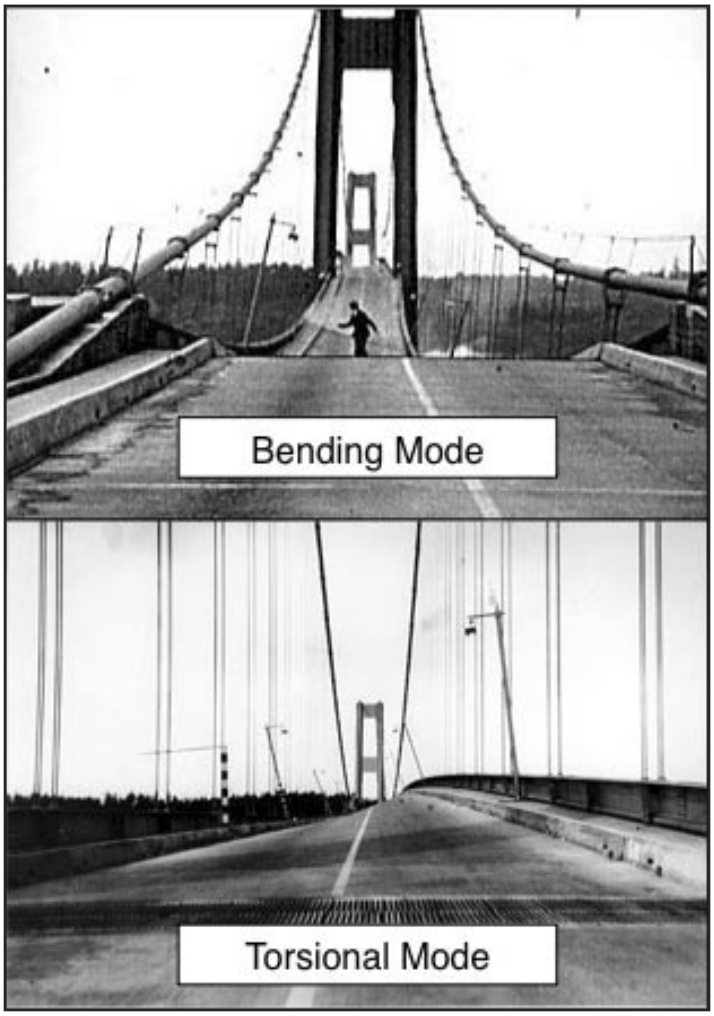



***Figure 1. Galloping Gertie's Bending and Torsion Modes***

## *Background*

In textbooks aeroelastic flutter is defined as "a dynamic instability associated with the interaction of aerodynamic, elastic and inertial forces." The essence of this definition involves understanding the interaction between an object and the surrounding air. Let's start with the simple aerodynamic concept of lift. In the case of Galloping Gertie, the bridge construction did not allow air to pass through the bridge; rather it was diverted above and below. This diversion of air creates lift and a pitching moment around the aerodynamic center. Due to the coupling between an increase in pitching moment and an increase in lift, a positive feedback loop is created. This means that the increase of one variable drives the increase of the other in an infinite loop. If not damped, the positive feedback loop leads to uncontrolled aeroelastic flutter and ultimate failure of the structure. In

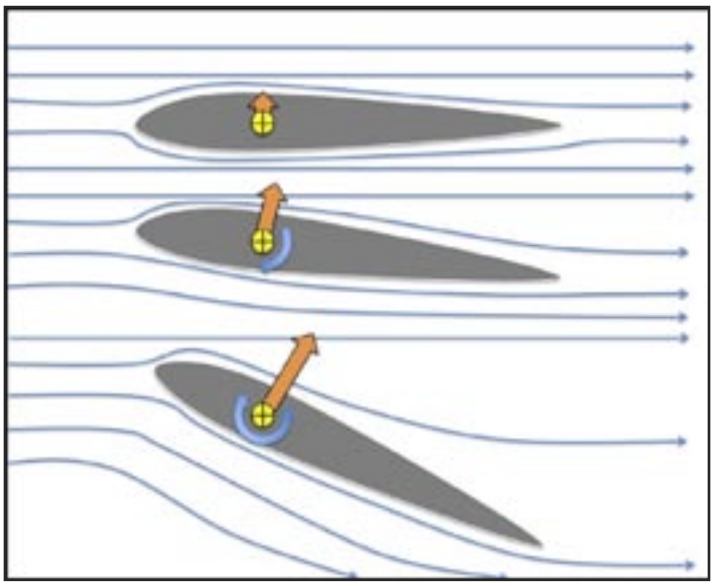


***Figure 2. Increasing Torsion on Pitching Wing***

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## How To Calculate Fin Flutter Speed

Figure 2 notice how an increase in the lifting force (orange arrow) creates a clockwise rotation of the wing and an increased torsional moment (blue arrow).

Unlike the Tacoma Narrow bridge, a fin attached to a rocket does not have large mechanical dampers. Instead, rockets need to rely on thoughtful construction and air to damp out any vibrational energy in the fins. Air is very efficient at reducing the amplitude of the vibration while the rocket remains under the flutter velocity. However, once the flutter velocity is exceeded the air will exponentially amplify the oscillations and rapidly increase the energy in the fin to the point of destruction. Figure 3 shows the exponential damping and amplification of vibrational energy in a rocket fin below and above the flutter speed. For the remainder of this article we will establish an equation for predicting the flutter boundary and discuss all variables involved.

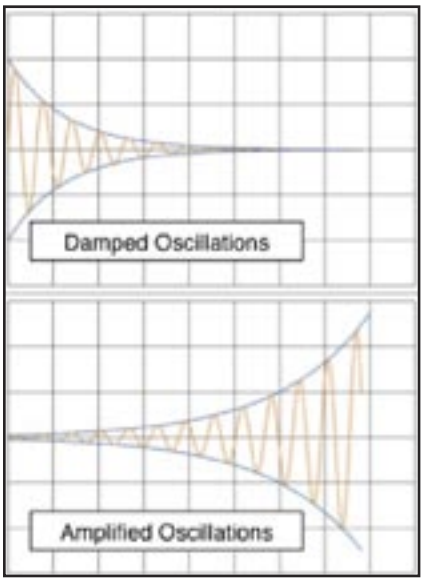



***Figure 3. Damped and Amplified Oscillations***

### *Flutter Boundary Equation*

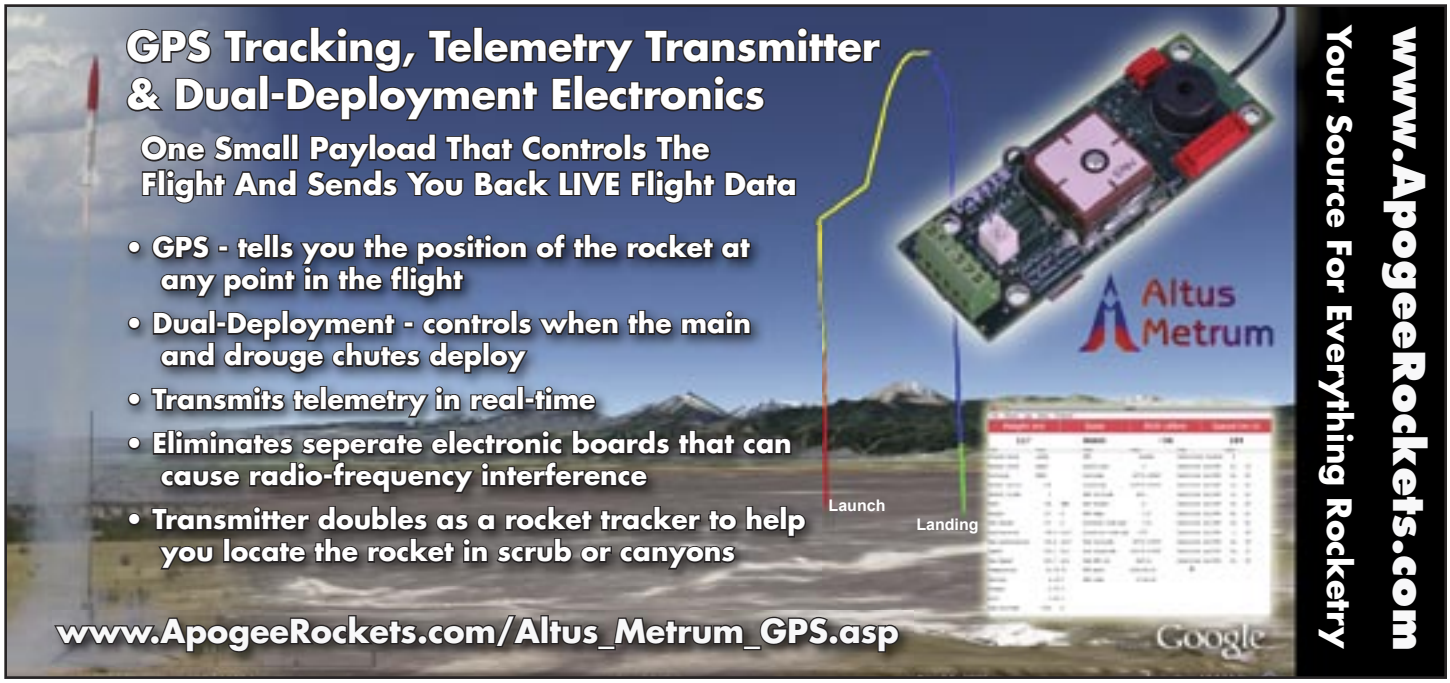
The Flutter Boundary Equation is based on an earlier calculation published in NACA Technical Paper 4197. If you are familiar with that paper you will notice that Equation 1 listed below is slightly different than the one presented in the technical paper. The most significant mathematical change is the use of a more accurate term for torsional modulus. This accuracy was gained by the inclusion of plate theory. Due to the complex nature of the flutter boundary equation we will focus our efforts on learning to understand the variables rather than trudging though the derivation.

$$V_f = a\sqrt{\frac{G}{\frac{1.337AR^3P(\lambda+1)}{2(AR+2)(\frac{t}{c})^3}}}$$

***Equation 1. Flutter Boundary Equation***

To begin our dissection of the Flutter Boundary Equation we will analyze the sole material property included in this equation, the Shear Modulus. Identified by the letter (G) it has units of pounds per square inch or PSI, and is the representation of the amount of deformation associated with a particular amount of force. Simply, the higher the Shear Modulus the more force it can handle.

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## How To Calculate Fin Flutter Speed

For the purposes of this equation the materials are assumed to be isotropic, which means that the mechanical properties of the material are the same in all coordinate directions. This assumption is very accurate for metals because they are manufactured in a relatively uniform way, but for wood and hand laid composites isotropy cannot be assumed. In wood, shear stresses are unique to each axis, making the material orthotropic. You have probably noticed how it is easier to split wood along the grain rather than trying to cut it perpendicularly.

Additionally, there is no guarantee that two pieces of wood, even from the same tree, will have the same material properties. The same is true for all hand laid composites as well, because of the variability in cloth fibers and epoxy application. Therefore, when using published shear data on orthotropic materials, add an additional safety factor.

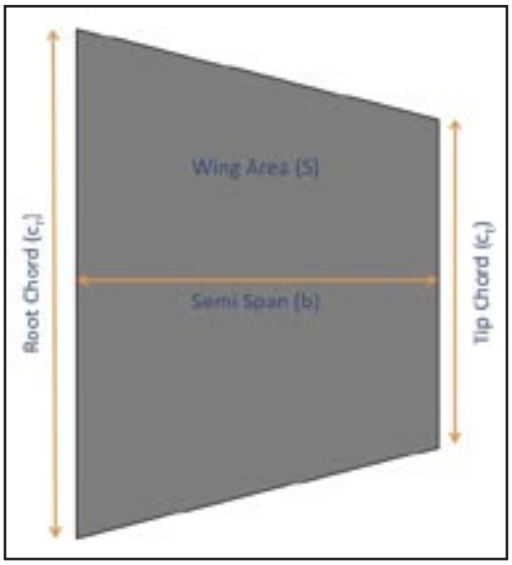



***Figure 4. Fin Geometry***

From Equation 1, the variables under the square root describing the geometry of the wing are the thickness of the wing (t), root chord (c), Aspect Ratio (AR) and the Taper Ratio (l). The equations for these variables are listed below, along with a wing geometry guide shown in Figure 4. All units should be in inches.

In a recent optimization study done by the Air Force they found that semi-span had the most impact in flutter speed calculations. Logically this makes sense, because a stubbier fin will be stiffer and more able to resist torsion as compared to a longer, more flexible fin. However, there is a trade off here with the minimum effective area needed to keep your rocket going straight. Through multiple design iterations using the RockSim software (www.ApogeeRockets.com/rocksim.asp), you should be able to come up with the right mixture of all desired values.

$$S = \frac{1}{2}(c_r + c_t)b$$

$$AR = \frac{b^2}{S}$$

$$\lambda = \frac{c_t}{c_r}$$

***Equations 2. Geometric Equations***

Next we come to Air Pressure (P). Using static atmo-

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spheric models you will find that in the Troposphere, which is below 36152 ft, temperature and pressure vary linearly with altitude according to those listed in Equations 3. There are more equations that model temperature and pressure changes in the Upper and Lower Stratosphere, which you can find at http://www.grc.nasa.gov/WWW/K-12/airplane/atmos.html. The answers to the pressure calculations need to be converted into pounds per square inch in order to make sure that all the units under the square root cancel out.

$$T(^{\circ}F) = 59 - .00356h$$

$$P(lbs/ft^2) = 2116 \times \left(\frac{T + 459.7}{518.6}\right)^{5.256}$$

***Equations 3. Temperature and Pressure Variations***

The last variable of the Flutter Boundary Equation is speed of sound (a). Dependent only on the temperature of the medium, the equation for the speed of sound is given in Equation 4.

$$a = \sqrt{1.4 \times 1716.59 \times (\mathrm{T}(^{\circ}\mathrm{F}) + 460)}$$

***Equation 4. Speed of Sound***

This equation already has the Ideal Gas Law constants associated with air inserted, making the temperature calculated through Equations 3 the only variable. The unit on this calculation is feet per second, which due to cancellation among all other units makes the Flutter Boundary Equation in terms of feet per second.

### *Equation Verification and Safety Factor*

In order to verify this equation, I have tested the Flutter Boundary Equation with data published in an article called *"Fin Flutter"* at http://www.info-central.org/?article=138 by Duncan McDonald. Although his attempt at calculating fin flutter was wrong (He forgot to add in the pressure terms and to keep consistent units. Also, some of the constants in the equation are deceiving because they are actually a combination of a bunch of constant terms), he had valuable test data from contributor Jeff Taylor who flew accelerometers in his rockets and recorded their maximum speed.

Based on the article's data, my Flutter Boundary Equation successfully predicted the two instances of flutter and the three safe flights of Jeff's rocket. That is a 100% success rate in five test cases. Without more significant testing the true accuracy of this equation will not be known, but preliminary calculations suggest that a comfortable safety margin is anything 20% below the flutter velocity speed. However, there have been instances of accuracy to within 5%.

The one major prediction problem is that the flutter velocity changes with altitude; therefore, to accurately predict flutter speed the altitude at which maximum velocity is achieved must be known. Usually this is not known, so keeping the rockets velocity under the maximum allowable at sea level is advised.

| | |
|---|---|
| Root Chord ($c_r$) | 9.75 in |
| Tip Chord ($c_t$) | 3.75 in |
| Thickness (t) | .125 in |
| Semi-Span (b) | 4.75 in |
| Shear Modulus (psi) | 380000psi |

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## How To Calculate Fin Flutter Speed

### *Example*

To truly cement your understanding of the Flutter Boundary Equation this following example will walk you through all equations necessary, with real numbers from an actual rocket. We're going to assume the rocket is at 3000ft.

Step 1: Preliminary Calculations

$$S = \frac{(9.75 \times 3.75)4.75}{2} = 32.06in^2$$

$$AR = \frac{4.75^2}{32.06} = .70$$

$$\lambda = \frac{3.75}{9.75} = .38$$

$$T = 59 - .00356(3000) = 48.32°F$$

$$P = \frac{2116}{144} \times \left( \frac{48.32 + 459.7}{518.6} \right)^{5.256} = 13.19 \frac{lb}{in^2}$$

$$a = \sqrt{1.4 \times 1716.59 \times (48.32 + 460)} = 1105.26 \frac{ft}{\sec}$$

Step 2: Plug into Flutter Boundary Equation

$$V_f = a \sqrt{\frac{G}{\frac{1.337 AR^3 P(\lambda + 1)}{2(AR + 2)(\frac{t}{c})^3}}}$$

$$V_f = 1105.26 \sqrt{\frac{380000}{\frac{1.337 \times .70^3 \times 13.19 \times (.38 + 1)}{2(.70 + 2)(\frac{.125}{9.75})^3}}}$$

$$V_f = 788.6 \frac{ft}{\sec} \cong 537.67 mph$$

The maximum velocity was measured at 449 mph, which is below the flutter speed. In another test with the same rocket, the maximum velocity was clocked at 631 mph. On that test the fins broke, as predicted by the equation.

### *Authors Note*

Firstly, I would like show my appreciation to Tim Van Milligan for publishing my article in the fantastic Peak of Flight Newsletter. Having been a subscriber for a long time now, I have always appreciated the constant stream of knowledge presented in these newsletters.

### *About the Author*

Zachary Howard is a recent graduate from Georgia Tech in Aerospace Engineering. From local launches to competing in the Team America Rocketry Challenge, his lifelong passion for rocketry has not wavered. After a recent failed Level 1 attempt, Zachary revisited his old textbooks and begun deciphering the phenomena of fin flutter that claimed his rocket.

