

A Contribution to the History of Planimeters – lesser-known instruments and variants

Detlef Zerfowski & John Vossepoel

Table of Contents

1. The challenge to structure the topic	1
2. Early usage of the term “Planimetry”	2
3. Bryon’s Planimetre Graphique, France, 1814 and 1826	2
4. Gelinski’s Calculateur, France, 1815	4
5. Zaręba’s (Zaremba) Planimeter, Poland, 1817	5
6. Laur’s Olarithme, France, 1819	7
7. Colberg’s Planimeter, Poland, 1820	8
8. Wießner’s mathematical instrument, Jena Germany, 1823 & 1828	10
9. Tenner’s Planimeter, Germany, 1828	12
10. Littmark’s Pollettcirkel, Sweden, 1831 & 1857	13
11. Stabrowsky’s Planimeter, Prussia, 1835	14
12. Lalanne’s Arithmoplanimètre, France, 1839	14
13. The Computing Scale, England, 1839	15
14. John Miller’s Pantoscale, 1841	16
15. Roulette de Dupuit, France, 1844 & 1878	17
16. Hennon’s Polygonometre, France, 1846	18
17. Godfray’s Agrometers, England, 1848	18
18. Källström’s Planimeter, Sweden, 1852	19
19. Gierer’s Planimeter, Germany, 1854	20
20. Burman’s Planimeter, Sweden, 1852 & 1856	20
21. Decher’s Planimeter, 1855	20
22. Stadler’s Hyperbole Planimeter, Austria, 1855	21
23. Buniakovsky’s (Bunyakovsky) Planimeter, Russia, 1855	22
24. Dasnoy’s Calculateur-Rapporteur, Belgium, 1856	23
25. Gangloff’s Planimeter, Bohemia, 1856	24
26. The Transformateur, the Netherlands, 1867.	25
27. Hofmann’s Instrument, Germany, 1874.	25
28. Ljungström’s Cirkelplanimeter, Sweden, 1870	26
29. Mechanised Harp Planimeters, Sweden, 1861 - ~1900	27
30. Kloth’s Hyperbolic Plates, Germany, 1883	31
31. von Erichsen’s Planimeter, USA, 1884	31
32. Updated timeline of early area measurement instruments	32
33. Bibliography	35

1. The challenge to structure the topic

In the Slide Rule GAZETTE 2021, one of us published a comprehensive overview [Zer2021-4] of early measuring instruments. In the meantime we have collected additional information for a second part on this topic. Tracing the development of these instruments in the 19th century presented many difficulties. So structuring this article was also a challenge. As a first approach, we tried to group the instruments according to the regions where they were invented and further improvements were made. This was particularly difficult because many of the instruments weren't widely used or even widely known (see, for example, the Swedish instruments mentioned below). However, we soon realised that for instruments that were more widespread, this regional categorisation no longer seemed to work.

Another approach to categorisation was by the underlying principle of the instrument. Again, we felt that this approach did not work well because some devices have overlapping principles and it was not possible to make a clear distinction between the inventions. Furthermore, due to the large number of devices invented, we wanted to pay special attention to early area measurements that had not been described in detail in previous

articles such as [DreHae2009-1, DreHae2011-1, Zer2021-4]. For this reason, we have decided not to use this categorisation scheme, which would have required us to re-examine the instruments from the articles cited above.

We have therefore chosen to present the inventions in chronological order, as summarized in the last section of this article. We will also try to provide information on the relationship between the various instrumental developments and their inventors.

2. Early usage of the term “Planimetry”

Before we start our journey into history of early instruments used for area measurement, let's have a closer look at the definition of the word planimetry. Planimétrie (or planimetry) is the study of plane measurements, including angles, distances and, in the particular context of this paper, surface areas. An early work on the subject is “La Geometrie Pratique” subtitled “La Trigonometrie Theorique & Pratique, la Longimétrie, la Planimetrie, & la Stereometrie” by the French mathematician Jacques Ozanam (1640-1718). Ozanam not only described conventional methods for calculating areas, but also introduced a graphical geometric transformation method, which would not be supported by mathematical instruments until more than 100 years later. The French cartographer and fortifications engineer Allain Manesson-Mallet (1630-1706) treated the subject of geometric transformations more extensively in the richly illustrated 4-volume work “La Geometrie Pratique” of 1702, with many worked examples in the chapter on planimetry [Mal1702].

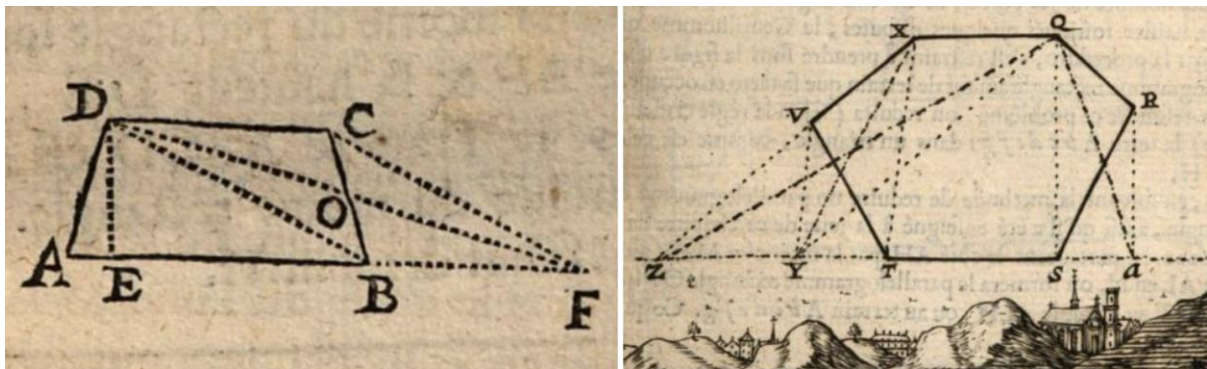


Figure 1: Geometric transformations. Left: graphically transforming a trapezoid into a triangle of equal area [Oza1684], p. 209. Right: transforming a more complicated polygon into a triangle of equal area, Manesson-Mallet, p. 233 [Mal1702].

3. Bryon’s Planimetre Graphique, France, 1814 and 1826

We will start our journey with the history of the early area measurements in the year 1814. Charles Henri Bryon, ingénieur géomètre en chef du cadastre, had been using the common method of dividing a polygon into triangles, measuring the height and base of each triangle, multiplying the products, adding the results and dividing by 2. Towards the end of 1809, aware of the advantage that a simple and rapid method could have over the known means, he began to experiment with the geometric transformation method to reduce the area of a polygon to the calculation of the area of a single triangle. We describe this approach in the section on Zaręba’s planimeter below.

As he continued his research, he noticed that if he transformed the triangle with an equal in area to the polygon into another right-angled triangle of the same size, with a fixed base and a variable height, he would always have 100 for one of the triangle’s dimensions, i.e. for one of the multiplication factors, which could then be done without using a pencil or even tables. He also felt that by giving the base of the triangle a fixed length, it was necessary to measure the height with great accuracy, which was made difficult by the small scale of 1:2500 generally used for land registry plans.

This led Bryon to choose a scale for calculating areas that was twice as large as that of the plan. To reduce an area to its real value, instead of multiplying the height of the triangle by half the base, it was necessary to multiply it only by 2. From this he concluded that, in order to always have 100 for one of the factors of the

product, it was necessary to give the triangles a base of 50 meters, a length which fitted most triangles in practice. In other words, when calculating the area of a polygon, a double transformation would simply reduce it to the measurement of a single line, which would have to be multiplied by 100.

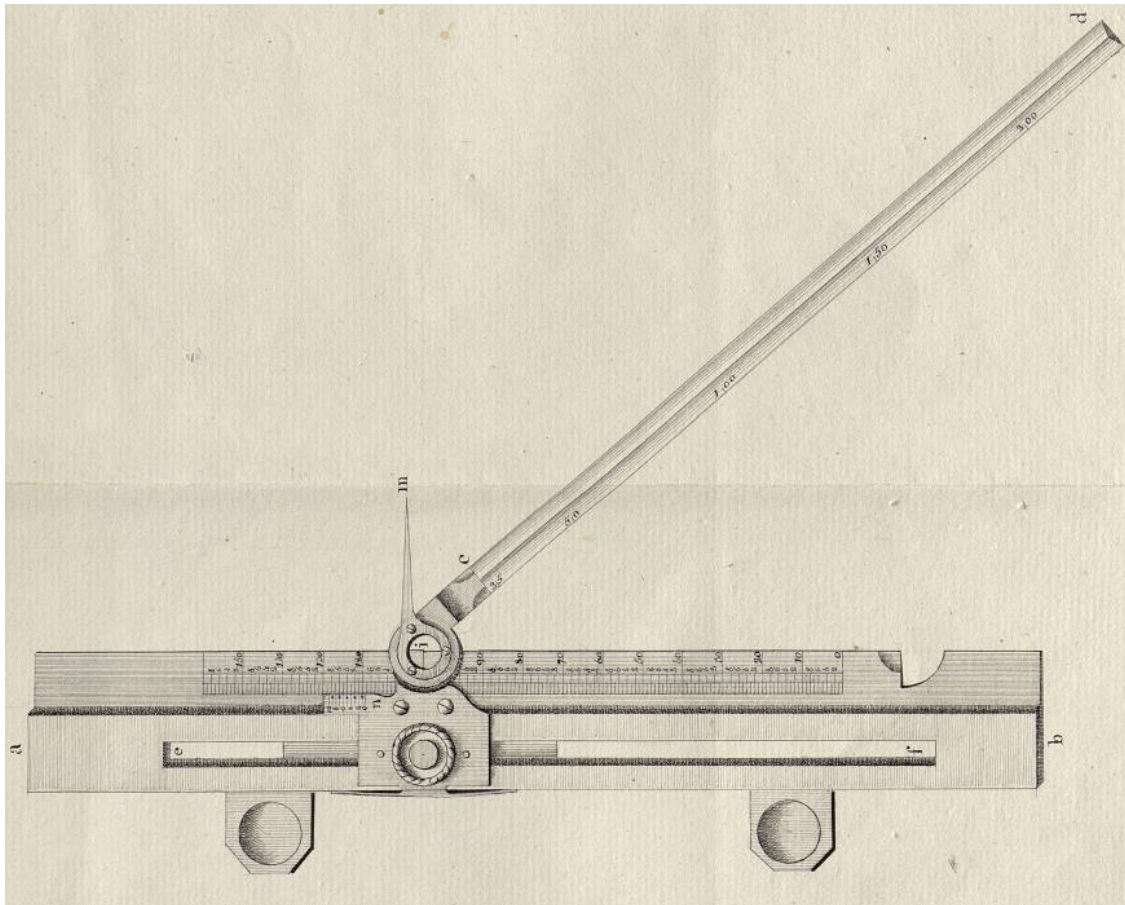


Figure 2: Bryon's Planimetre Graphique from [Bry1814] (Courtesy of L'Héritage des Ponts et Chaussées).

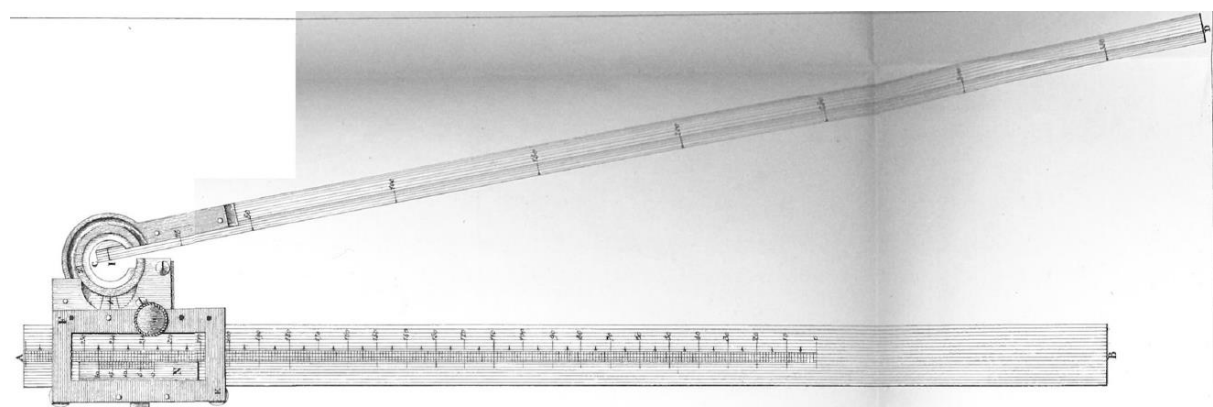


Figure 3: Bryon's Planimetre Graphique variant from 1826 [Bry1826] (Courtesy of Biblioteca e Arquivo Histórico da Economia).

To make the transformations easier, Bryon devised an instrument which he called a Planimètre Graphique and published a treatise on its use in 1814 [Bry1814]. However, it was necessary to mark the intersection of the parallels with the base with a pencil line on the base, and Bryon noted that there was a risk of confusing the last mark with one of the previous ones. This prompted Bryon to create a new model in 1826 that

avoided the need to mark the baseline, and without having to draw a single line other than that of the triangle equivalent to the polygon [Bry1826]. Apart from references to Bryon's treatises of 1814 and 1826 in various bibliographies, we couldn't find any other references to his planimeter in the literature. In fact, later inventors seem to have been unaware of Bryon's planimetre graphique, as it is never mentioned, leading us to the cautious conclusion that it must have been known and used only locally.

4. Gelinski's Calculateur, France, 1815

Frederic Charles Gelinski, was a Polish nobleman born 10 August 1771 in Wilna, Poland. Fighting against the French during the French Revolutionary Wars he was taken prisoner in 1792 with his Austrian regiment. He remained in France and was admitted to the Ponts et Chaussées in Troyes, then, in 1812, to Holland, then controlled by France. Naturalized in 1814 he became head of the Cadastre in Angers [NN1815] and was sent in disgrace to Digne following the July Revolution of 1830. He died there on 18 October 1837 [NN1983].

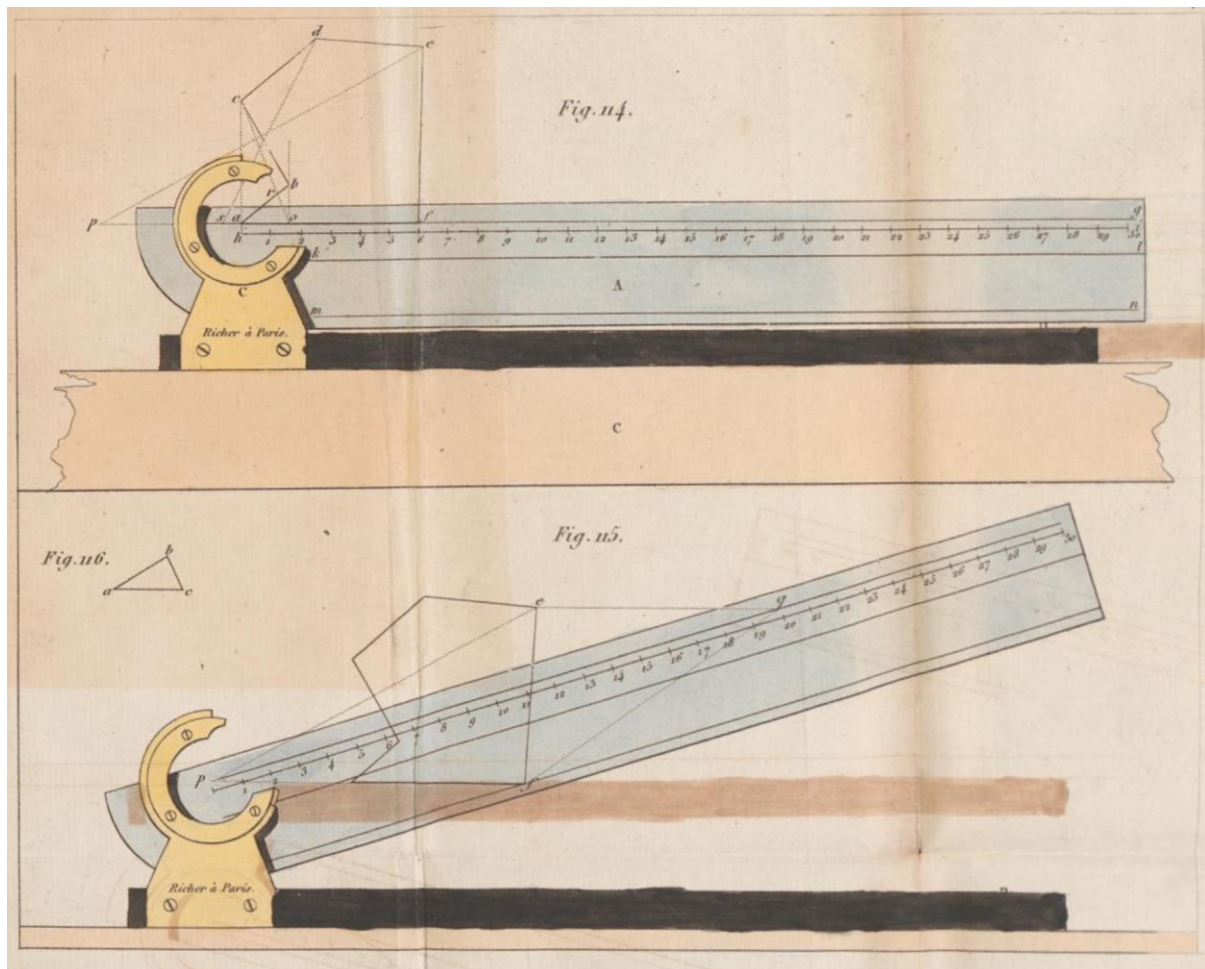


Figure 4: Gelinski's Calculateur from [Lec1846].

Gelinski devised an instrument that facilitates the transformation of an irregular polygon to its equivalent triangle by constructing parallel lines. The area can then easily be calculated from the resultant triangle. Gelinski described its usage in a pamphlet published in [Gel1815]. Gelinski's pamphlet is listed in several bibliographies but we have been unable to locate a copy in any library. However, there is a ZVAB bookseller in France who has, or had, an undated handwritten and illustrated copy of Gelinski's treatise.

Gelinski's Calculateur was considered to be sufficiently useful to be treated in several popular 19th century French surveying textbooks. One is by Foucard and Place [FouPla1838], another being by François Lecoy [Lec1846] and from the Netherlands in [NN1850-1] as well.

Although Gelinski may not have been aware of Bryon's planimeter, as he makes no mention of it, his instrument is similar in construction. Two years after Gelinski's idea, Zaręba's device would also embody the same approach. The geometrical principle is explained in the next section.

5. Zaręba's (Zaremba) Planimeter, Poland, 1817

In his book [Zar1829] Zaręba¹ summarized the existing approaches by Zobel [MulZob1815], Colberg [Col1825], Wagner [Wag1821], Possener [Pos1823] and Wießner [Wie1828] and evaluated all of them as insufficient. In his introduction², Zaręba mentioned:

"The methods of calculating the geometric plans, which have been practised until now, as tedious and time-consuming, have given me reason to look for other, more appropriate circumstances. Not having any languages other than my mother tongue, I could not make use of the inventions in this matter described in a foreign language; and therefore I did not dare make my own known to the public, remaining in constant uncertainty as to whether my methods were not a repetition of those described in the foreign language. Methods are not a repetition of those already known."

While constantly engaged in the practice of geometric measurements and visiting various localities, I came across people to whom I had to explain my methods facilitating geometric calculations in order not only to satisfy their curiosity but also to gauge their opinion on the usefulness of the invention. I was able to compare and contrast such opinions, as I found more in favour of the usefulness of the invention than against; I did not, at the same time, oppose the insistence and assurances of good effect of the experts and experts in mathematical skill; and, above all, I took advantage of the assistance of His Excellency Duke Adam Voivode of Warsaw. Duke Adam Voivode Czartoryski, I dared to ask the competent Government Authorities to issue me with an official certificate of the novelty and usefulness of my invention, as well as for permission to publish the dissertation in print and dedicate it to my co-workers for their convenience."

My requests received the desired result. I therefore grant to the co-operators both the invention for which I have been granted, and the confirmation of the Government which I have received, which will serve as a kind of prior assurance that my tool can be used to the advantage of my colleagues. My tool can be used profitably."

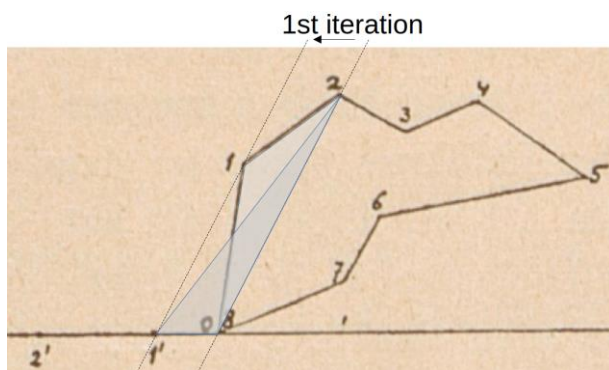


Figure 5: Zaręba's approach: 1st iteration moving point 1 to 1'.

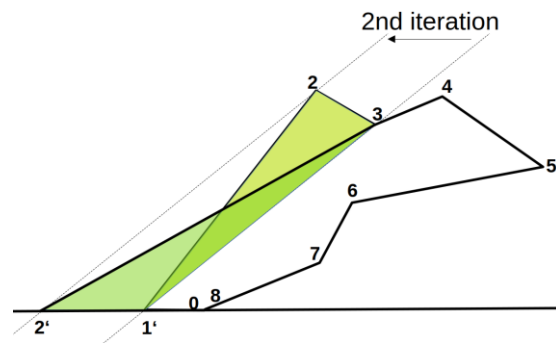


Figure 6: 2nd iteration: Moving 2 to 2' - the 8-polygon becomes a 7-polygon of identical size.

Interestingly on page VII of [Zar1829], Zaręba mentions that on 11th March 1817 he was granted the invention for 10 years for the whole Kingdom of Poland. Since the sovereign patent legislation operated in the Kingdom of Poland from 1817 to 1867 [Dol2018], Zaręba's patent must be one of the first Polish patents. Therefore, although we don't have any further evidence of the granted patent, we can date Zaręba's planimeter to 1817. In addition, according to the note in [NN1829-3], Zaręba renewed his patent in 1829.

Zaręba gives a detailed description of his invention. Although the method of graphically transforming a closed polygon into a triangle of equal area was known much earlier (see section 2 and [Mal1702]), the method is quite remarkable. We will now explain how it is done using Zaręba's device. He transforms an arbitrary closed polygon with n corners into a polygon with $n-1$ corners (of the same area) by geometrically moving one corner to a horizontal base line. Again we emphasize that by the geometrical construction the $n-1$ -polygon has

¹Sometimes Zaręba is called Zaremba, because of the Polish pronunciation of the name Zaręba.

²Translated by DeepL with some edits.

the same area as the original n -polygon. The same process is applied to the $n-1$ -polygon resulting in an $n-2$ -polygon. Again the $n-2$ -polygon is of identical size as the $n-1$ -polygon, which is of same size as the original n -polygon. This process continues until the construction results in a triangle (which is of the same area as the original n -polygon). Determining the area of the final triangle therefore gives the area of the original n -polygon.

Figure 5 shows an 8-polygon from [Gra1908-1] explaining Zaręba's approach. Consider the closed polygon with 8 corners, which area is to be measured. A horizontal line is drawn crossing point 0. Next a diagonal line through points 0 and 2 is taken. Now this line is parallel shifted through point 1. This line intersects with the horizontal line at $1'$. The core idea is that the triangles 012 and $01'2$ have the same area; because both triangles have the same base 02 and the same height (the distance between the two parallel lines). As a consequence the closed 8-polygon 012345678 is of identical size to the polygon $01'2345678$. The next iteration (Figure 6) starts with the polygon $01'2345678$, the diagonal $1'3$ is taken. This diagonal is parallel shifted through point 2, intersecting the horizontal line at $2'$. Again the triangles $1'23$ and $1'2'3$ are of same size. Consequently the polygons $01'2345678$ and $01'2'345678$ have the same size. But, be aware, the polygon $01'2'345678$ does not have 8 corners any more – there are only 7 corners left (since the points 0, $1'$ and $2'$ are laying on the horizontal line). Iterating the process for each point 4, 5, 6 and 7 reduces the number of polygon corners by one, finally leading to a triangle with identical size of the original polygon. The size of the final triangle can be determined by measuring the base and the height.

With this knowledge we are now able to understand Zaręba's device. It graphically supports determining the intersection points $1'$, $2'$, ... on the horizontal line.

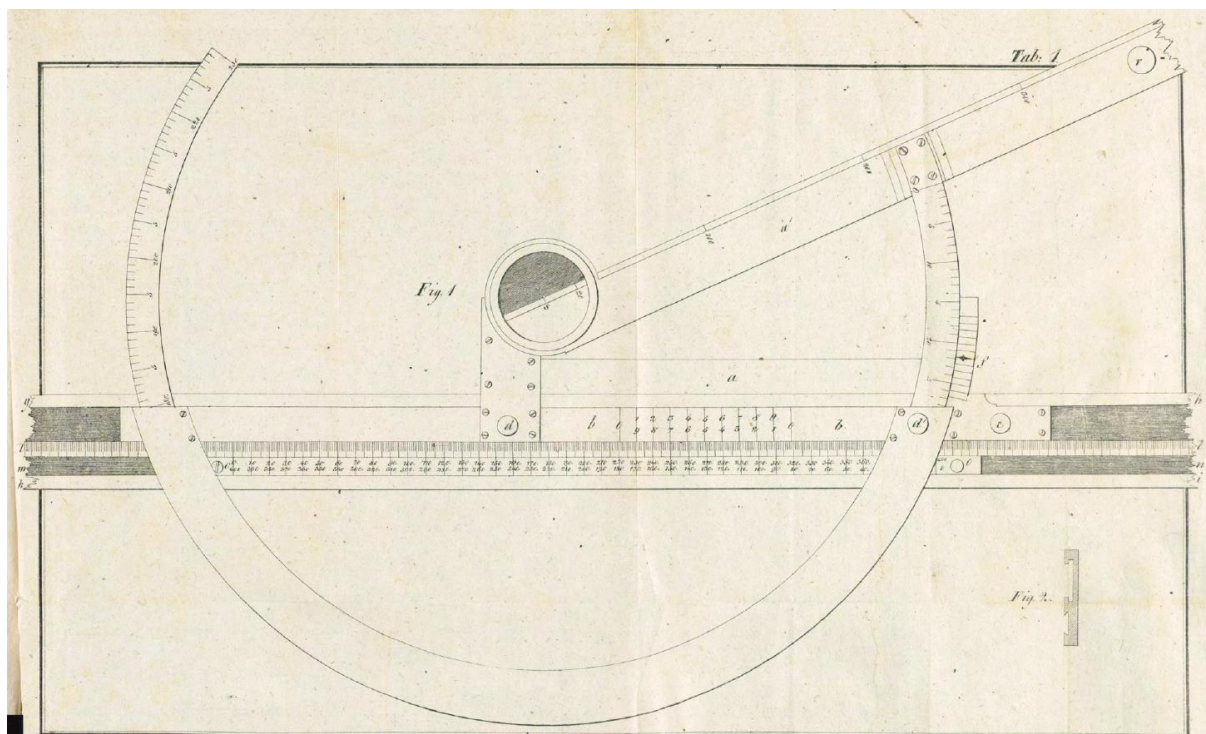


Figure 7: Zaręba's planimeter from [Zar1829]. Compare the similarity with Gelinski's device in Figure 4.

The device is placed below a figure to be measured and consists of a horizontal ruler carrying an unnumbered scale. Inside the ruler two sliding scales can be moved and afterwards fixed by screws. The lower sliding rule carries the index numbers for the unnumbered rule. The upper sliding rule provides a Vernier. On this upper sliding rule at point d a protractor (a and a') is mounted. The rotatable arm a' is used to be the diagonals (e.g. 02 in Figure 5, or $1'3$ in Figure 6). After positioning the arm as such a diagonal, one moves the upper sliding rule horizontally, which realizes the parallel shift of the diagonal. This procedure is iteratively applied as above described. In the final step a rectangular triangle has been constructed, which area is easily determined with the help of the index numbers on the lower scale. For a more detailed description of the usage of Zaręba's planimeter we refer to [Gra1908-1, Gra1908-2].

The fact that the Polish inventions have been hardly known outside Poland has also been addressed in the Austrian journal on surveying by Grabowski [Gra1908-1, Gra1908-2]. In the article Grabowski refers to a discussion on the priority of the planimeter inventions of Gangloff and Schlesinger in [Mul1879]. In the referred article Müller allocated the priority of the invention to Gangloff based on his publication in 1856 [Gan1857].

Grabowski did not agree with this statement and pointed out that Gangloff's planimeter was identical to the planimeter invented by Jan Zaręba in 1829. There is only a small mechanical difference between the two instruments. Zaręba used locking screws to prevent unintentional rotation of the arm a' . Gangloff used a scale mounted on a glass plate for the same purpose. Grabowski noted that Gangloff's construction was more prone to errors than Zaręba's planimeter because of the reading through the glass plate.

6. Laur's Olarithme, France, 1819

Géomètre Jean-Antoine Laur (born 1785) introduced a planimeter, which he called the Olarithme, in the first edition of his *Géodésie Pratique*, published in 1819. This popular French surveying textbook went through many editions, the 6th being published in 1855 [Lau1855] and a 2-volume German edition in 1857 [Lau1857-1, Lau1857-2]. Laur presented the Olarithme to the French Academy of Sciences in 1821. He continued to extol the merits of his Olarithme in his *Notice sur les Planimètres* published in 1859 [Lau1859].

Laur participated in the Great Exhibition of 1851 and received an honorable mention for his Olarithme. Favaro, quoting a report of the jury, gave the following description of the Olarithme

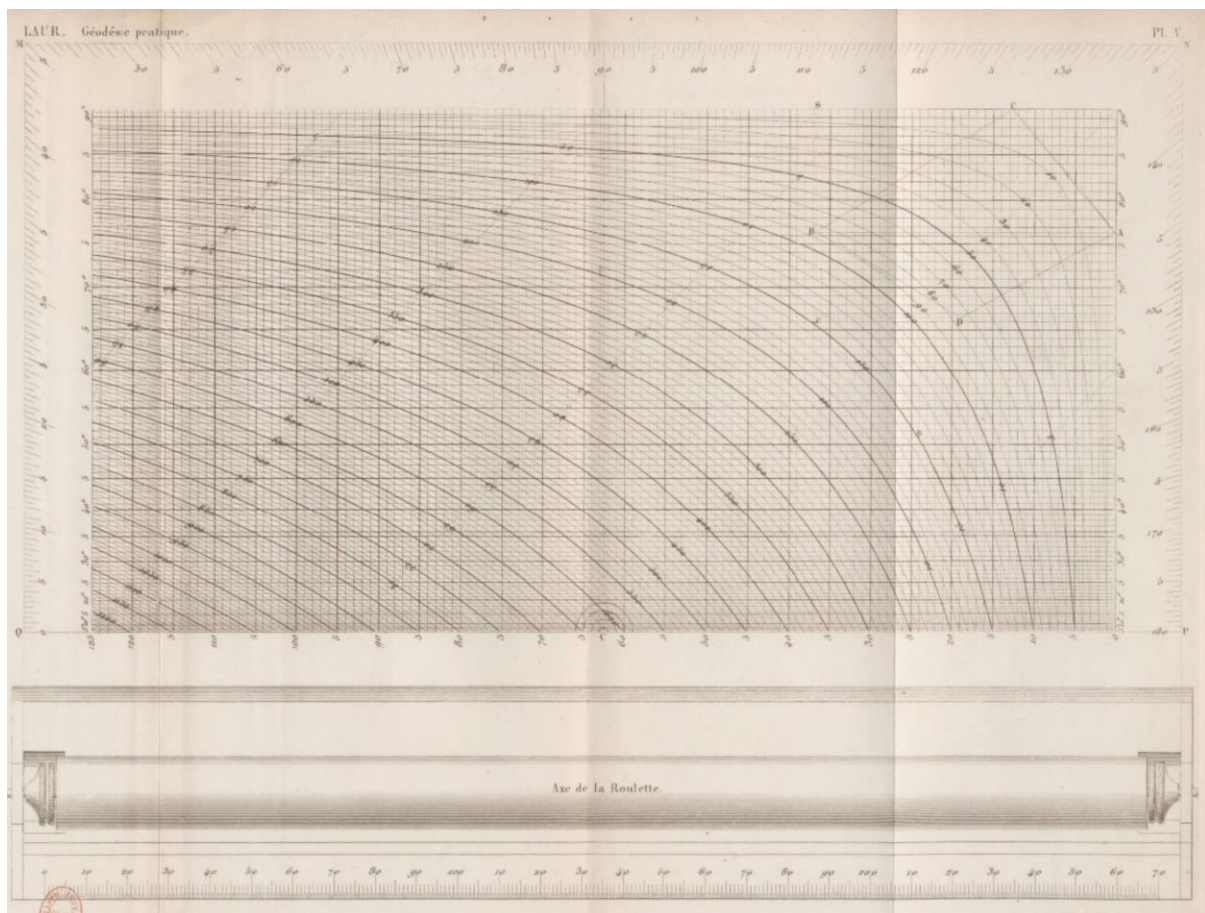


Figure 8: Laur's Olarithme

“Laur (France, Nr. 567) has exhibited a planimeter in which the area of a plane, supposed to consist of triangles, is first reduced, by geographical construction, to any right-angled triangle. On a rectangle of either gelatin paper or transparent horn are engraved a series of hyperbolas, such that the product of the ordinate and abscissa contained for each, shall change from hyperbola to hyperbola by a constant increment. The extremity of one side of the triangle being made to coincide with the common centre of all the hyperbolas, and that side

with the asymptote, the opposite angle will of necessity fall on one or between two adjacent ones of the hyperbolas, and its area (or its double) will be there-fore expressed by the ordinal number of the hyperbola, or the number of the lowest plus or proportional part for the distance between that and the angle, as compared with the interval between the two consecutive curves..." [Fav1873].

7. Colberg's Planimeter, Poland, 1820

Let us now look at the inventors in Poland. An overview of Polish planimeters had been published by Feliks Kucharzewski in 1902 [Kuc1902]. The Polish instruments have not spread much beyond the borders of the region. For this reason, there are very few articles on the devices and references to them outside the Polish language. A detailed description of Julius Colberg's device has already been given in [DreHae2009-1], but without listing any corresponding references. Colberg's original Polish [Col1824] and German [Col1825] publications give details of the invention. Christoph Julius Heinrich Colberg (also spelled Kolberg, or in Polish Krzysztof Juliusz Henryk Kolberg) was born on 7 July 1776 in Woldegk and died on 5 September 1831 in Warsaw.

Colberg was aware of Zobel's and Wagner's planimeter and referred to their publications from 1815 and 1821. He left it up to others to compare the performance of Zobel's and Wagner's devices with his. To

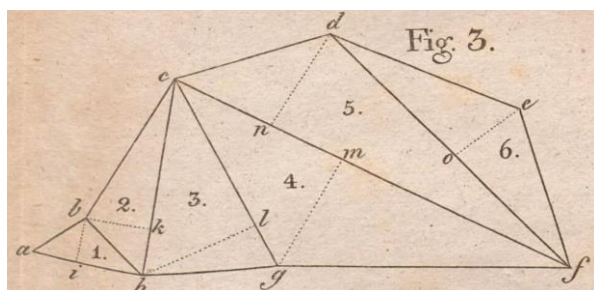


Figure 9: Closed Polygon abcdefgh divided into six triangles ([Col1825] Tab. II).

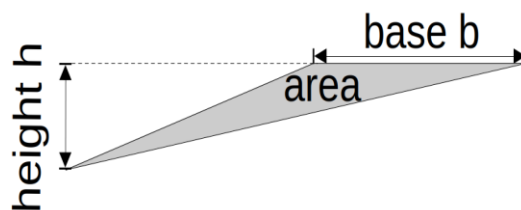


Figure 10: Area of a triangle:
 $\text{area} = \text{base } b * \text{height } h / 2.$

claim the date for his invention Colberg referred to an article in a Polish forestry journal from 1820, where he originally published his device [Col1820-1].

Educated at the Technical University of Berlin, Juliusz Colberg became Professor of Surveying at the Royal University of Warsaw in 1817. He was a teacher at a local forestry school and an active member of the Royal Society of Friends of Science in Warsaw. In 1820 he was awarded a Master of Fine Arts and a Doctor of Philosophy by the University of Warsaw for his work in cartography and the construction of his planimeter. He also translated Polish poetry.

Colberg's approach of determining areas of closed polygons is based on dividing the area into triangles. The areas of these triangles are determined and summed up, leading to the total area enclosed by the polygon. To understand the approach we need to remember our trigonometry courses at school: The area of an arbitrary triangle is calculated by the base times the height divided by two.

Instead of calculating the rectangle area $b \times h$, Colberg applied the principle of rectangle squaring, i.e. the area of a rectangle $b \times h$ equals the area of a square with side length calculated by the geometric mean

$$a = \sqrt{(b * h)}$$

Therefore the area of the triangle is $(a * a) / 2$. Colberg's approach is to find the geometric mean of the base and height of any arbitrary triangle. With the geometric mean the area of the triangle can immediately be read from a corresponding multiplication table (Figure 11) which Colberg calls planimeter. The results in the table are in the old plot area units of "Morgen" and "Quadrat-Ruthen" (square rods), where 1 Morgen equals 180 square rods. In the multiplication lattice the large, bold numbers represent the Morgen and in the cells right to this number the additional square rods are given. A more detailed look at the multiplication table shows that the table consists of two parts, the upper lattice in Figure 11 and the lower lattice above the proportional circle. This lower lattice is the continuation of the upper lattice at its right end. So the complete lattice consists of nine horizontal lines plus one skew line at the bottom. Additionally there are 101 vertical lines (dividing the table into 100 columns).

Looking up the result $(a * a) / 2$ for a given geometric mean "a", one needs to look at corresponding cell. Moving 1 cell up increases the geometric mean "a" by 10 rods. Moving one cell to the right increases "a"

by 1/10 rods. Be aware moving from the first column to the last column leads to an increase of $100 * 1/10$ rods = 10 rods. This is the reason for the skewed line starting at left at zero and ending at the right with a distance to the first horizontal row representing 10 rods.

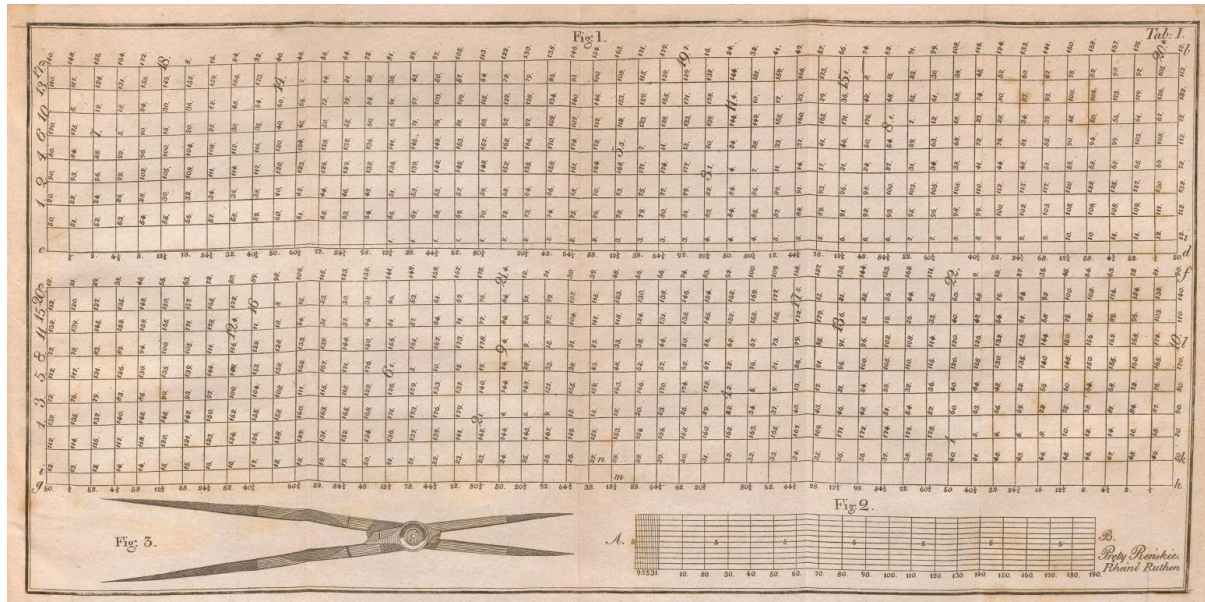


Figure 11: Colberg's multiplication table for squaring the geometric mean from [Col1825].

As an example, let us search the result for $(a * a) / 2$ for $a = 53.2$ rods. For this we need to look right of column 32 (32 / 10 rods) (be aware the first column represents 0/10 rods) and up above the 5th row representing 50 rods (be aware the 1st row represents 0 rods). The corresponding table entry shows 155 (square rods) and the first large number to the left is a 7 (Morgen). Additionally, below the corresponding column a small 12 is noted indicating 12/100 square rods. In total the area of triangle with the geometric mean “a” equals to

$$(a * a) / 2 = 53.2 \times 53.2 / 2 \text{ square rods} = 1415 \text{ square rods} = 7 \text{ Morgen } 155.12 \text{ square rods.}$$

Although Colberg explained that the results of his planimeter (the multiplication table) would only be reliable if the device was made entirely of brass with precisely engraved lines. Printed on paper, the results of the planimeter would vary greatly due to changes in the temperature and humidity of the paper.

After the description of above calculations, the geometric mean of the base and the height of triangles need to be determined. Colberg describes a geometric procedure using a reduction compass (see lower left in Figure 11), which physically reduces the distances between the tips of the longer compass arms to one half between the tips of the shorter compass arms. First Colberg puts the base and the height of the triangle on the horizontal line (see Figure 12). With the reduction compass he gets the arithmetic mean $a = (b+h)/2$ between the tips of the shorter compass arms. Taking the distance a from the endpoint b (or endpoint h) one compass tip is now in the centre between the two end points of b and h. Without changing the compass he takes a circle around this centre point to determine the intersection with the orthogonal axes. The distance m between this intersection and horizontal axes is the geometric mean: $m = \sqrt{(b * h)}$. Taking this distance in the compass the value for m is read with the scale (see bottom right in Figure 11). This is followed by the multiplication described above.

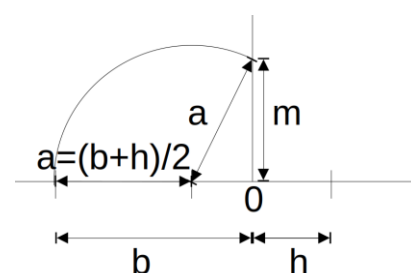


Figure 12: Geometric determination of geometric mean of h and b.

8. Wießner's mathematical instrument, Jena Germany, 1823 & 1828

From an early age Johann Gottfried Wießner had a special affinity for mathematics. He gives an account of his early years in the introduction of his treatise on parallel lines in 1833 [Wie1833]. In 1809, after

Wießner was already familiar with Zobel's and Müller's planimeters when he published his treatise in 1823, and wishing to create an instrument more suitable for Saxe-Weimar, where the units of area were not decimal, he set about designing a calculating clock that would make area calculations easier. In the beginning of 1828 Wießner constructed a model of a calculating clock with wooden cogwheels, which he described in "Beschreibung einer mathematischen Rechningsuhr" (Figure 14) published in the same year [Wie1828]. On the title page Wießner gives this concise description: "*By the use of this clock the whole calculation is*

accomplished, because it adds everything, divides every line by 2 and multiplies all figures, both triangles and trapeziums from 1 to 200 feet, or 20 Paris lines in height."

The fact that Wießner had to deal with non-decimal units of area meant that his clock had to be more complicated than other calculating devices. The Paris line is an old unit of length. 12 Paris lines are equal to one Paris inch (pouce) which is longer than the English inch. In 1799, the unit Paris line was used to determine the length of the standard meter, which is equal to 443.296 Paris lines. One Paris line is therefore equal to 2.2558 mm. With his calculating clock, Wießner was able to determine triangles and trapezes up to 20 Paris lines (4.5 cm) in height.

Figure 14 shows the detailed construction. Fig. II within this shows the device with its three dials:

- the large dial is divided into 100 decimal feet (we assume Paris feet) (equals 1 square ruthe (Engl. rod)
- the dial divided by 35 ruthen (equals a quarter acre)
- the dial divided by 12 complete acre.

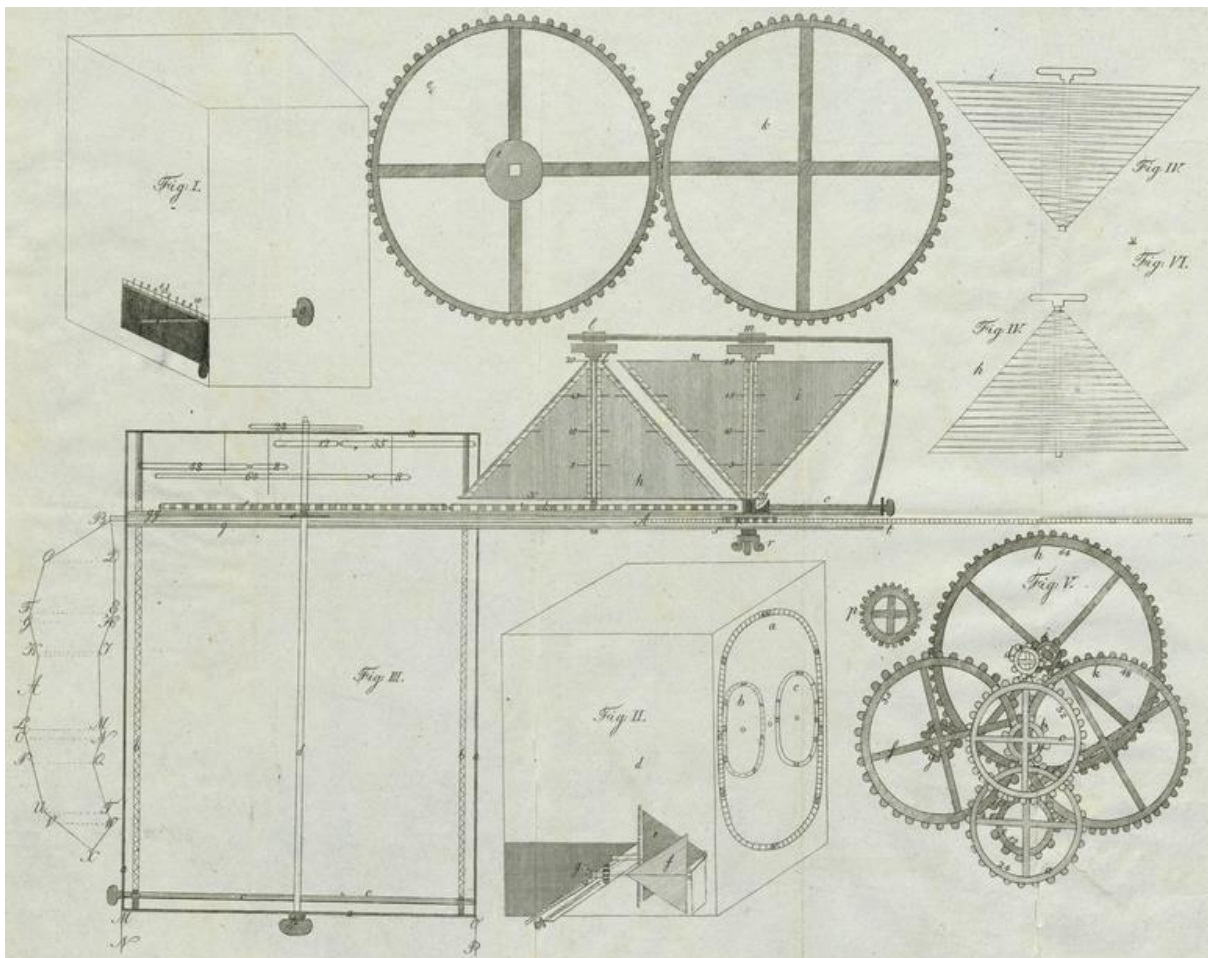


Figure 14: Wießner's Rechenuhr (engl. calculating clock) from [Wie1828] (Courtesy Universitätsbibliothek Tübingen, Signature Bc 114).

When dial c) makes one complete turn (12 acre), the dial b) will turn 48 times (48 quarter acre = 12 acre) and dial a) will turn 1680 times (48 quarter acre = 48×35 ruthen = 1680 ruthen). Fig. V in Figure 14 shows the gear construction for the corresponding transmissions. Behind the gear construction, Wießner designed a multiplication system based on two identical, but opposite multiplication cones (centre of Figure 14). The cones have a circumference of 120 Paris lines (270.7 mm) at the base (corresponding to a diameter of 8.62 cm) and 6 Paris lines (approx. 13.58 mm) at the top (corresponding to a diameter of approx. 4.3 mm). Both cones are connected by a fine chain that runs around both cones. Positioning the chain around the bottom of the left cone and the top of the right cone gives a transmission ratio of 1:20. Moving the chain to the top of the left

cone and therefore at the bottom of the right cone gives a reverse transmission ratio of 20:1 is in use. Multiplication is achieved with the different cone ratios.

It should be noted, that the cones described do not have teeth to carry the chain. The cones are hollow and have four slots on their surfaces from the bottom to the top. Inside the hollow cones there are two additional cones (Fig. IV in Figure 14). On the surface of the inner cones, a screw thread runs around. In this screw thread on both inner cones 4 small forks can run. These four forks are guided in the four slots of the outer cones. The chain is driven by the four forks of both cones. So the transmission ratio between the two cone systems can be changed by rotating the inner cones against the outer cones, causing the forks in the slots of the outer cones to move between the bottom and top of the two cone systems. In modern terms we would call this a cone chain transmission.

To measure the area (see in Figure 14 at the left of Fig. III), the height of the trapezoids (distance between the parallel sides,) is entered with the knob (bottom of Fig. III), which causes the inner cones to rotate and thus changes the transmission. The length of the parallel lines of the trapezoids is measured with the other knob (bottom left in Fig. III). The final result is displayed on the dials on the front of the unit.

Although Wießner encouraged others to copy his calculating clock, no copy is known to have been made, nor does the original Wießner calculating clock appear to have survived. This may have been due to changes in units of measurement that made it obsolete, or it may simply have been found to be impractical to use. In 1835 Wießner published a textbook on surveying entitled “Praktische Anweisung zur Feldmeßkunst” [Wie1835]. In the chapter on calculating the area of land plots, he curiously makes no mention of his mathematical instrument or his calculating clock.

9. Tenner’s Planimeter, Germany, 1828

In the course of our research, another unknown (at least to the authors) instrument for measuring surface area turned up in the literature. In the German journal “Didastalia” [NN1829-2] a device invented by Tenner is mentioned. In [Ten1828] Tenner described how to determine plot areas avoiding any calculation. On the right in five adjacent plots (1 to 5) are shown. The first plot (ABcd) is divided by a diagonal into two separate triangles (ABc) and (Acd). Remember again Figure 10. The area of a triangle is calculated by the base times the height divided by two. Applied to the triangles in Tenner’s plot 1, the area of (ABc) calculates to distance Ae times distance Bc divided by 2 and the area (Acd) equals Ad times cf divided by 2. Therefore, area of plot 1 equals to $((Bc \times Ae) + (Ad \times cf)) / 2$

Tenner avoided the arithmetical multiplications by performing graphical multiplications. For this, he drew a set of similar triangles (left in Figure 15). He put equidistant marks $p, p', \dots$ on the ordinate and drew lines through these marks and the point C. Note: $p' = 2 \times p$, $p'' = 3 \times p$, etc. Based on the map scale with the figure to be measured, the distance Ap represents a real distance in the field. This value has been written next to the mark p. Since $p', p'', \dots$ are multiples of p, corresponding multiples of the real distance are note to the corresponding marks $p', p'', \dots$. On this ordinate the areas of the plot triangles will be read. The abscissa is divided into equidistant portions, starting from 1 at point A, 2 in the middle between A and C and corresponding intermediate values in between. At corresponding abscissa marks vertical lines are drawn. The determination of the plot triangles one had to take the measured width of a plot (e.g. Bc) as the base, chose the corresponding abscissa mark. The triangle height (in this case Ae) is taken by a circle. Afterwards one end of the circle is put in the chosen abscissa mark and with the other end the distance on the corresponding vertical line is marked. A line from C through this point will cut the ordinate at certain point. The corresponding value on the ordinate (divided by 2) gives the area of the triangle. The division by 2, can already be considered in the design by scaling the ordinate values by a factor of two. Tenner called these kind of “multiplication graphs” as “Inhaltsmesser” (Engl. content meter). Obviously several graphs for different ranges of plot width and map scales are required and given in [Ten1828]. He asked the readers to draw these “Inhaltsmesser” by themselves according to their needs. At this point of his treatise Tenner proposed to combine many different ordinate scales leading to hyperbolic curves engraved on a glass plate. With this device no circle is required any more. The glass plate could be moved over the figures to be measured and the results could be read directly without taking any intermediate measurement. This is an early description of a simple hyperbolic table.

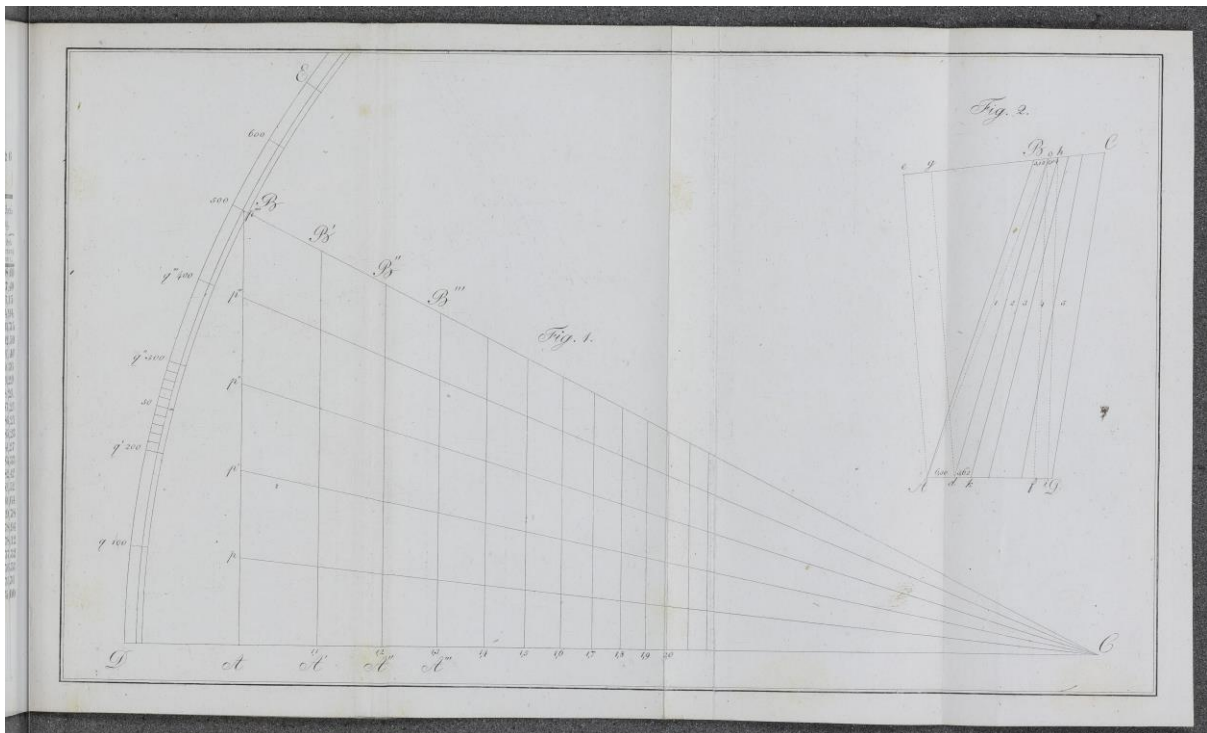


Figure 15: Tenner's approach from [Ten1828], courtesy Universitäts- und Landesbibliothek Darmstadt.

Tenner also created a mechanical aid to help calculate the above. A rotatable scale, fixed in point C, can be used to read the multiplication results on the ordinate. To make it even more convenient, the results to be read are written on the circular arc (Figure 6 Figure 25 on the left the arc BD). From this point Tenner called his device a calculating disc and explained also how it can be used for divisions and determining square roots. The treatise did not show any device. Since Tenner had the intention, that the readers create devices by themselves and the fact that we did not find any further reference to his device or even his treatise, we assume that the instrument he described has not been successful in the market, at least not under his name.

10. Littmark's Pollettcirkel, Sweden, 1831 & 1857

Fredrik Littmarck (1801-1876), land-surveyor, son of mining-surveyor Jonas Littmarck (1758-1847) [Eks1903, p.91]. At that time a common method in Sweden was to place a glass plate with an engraved rectangular grid over an enclosure and simply tally up the number of squares [And1876, p.223]. Littmarck "endeavoured to find a shortcut by which one could reach the goal more easily", devised an instrument which he called the Pollettcirkel, and in 1831 published a pamphlet describing its use [Lit1831]. Similar in appearance to a beam compass, the Pollettcirkel consisted of a wooden beam on 2 brass supports, and a movable cursor. The Pollettcirkel would be placed over the enclosure on top of a glass plate grid or tracing paper with equidistant lines. After measuring the first strip with the cursor, keeping the cursor in that position, the Pollettcirkel is then moved in its entirety to the next strip, with the cursor now at the beginning of the next strip. This process is repeated until all the strips have been measured and the sum of all the strips is read off the scale. To obtain the area, the total length is simply multiplied by the common line width.

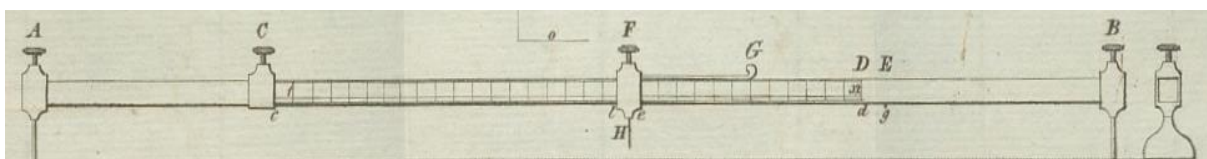


Figure 16: Pollettcirkel from [Lit1831] (Courtesy of Kungliga Biblioteket).

In 1856 Littmarck received a patent for an improved version of his Pollettcirkel, and published on pamphlet on its use [Lit1857]. Although not illustrated in this pamphlet, the main difference from the previous model is that the cursor/pointer has been replaced by a brass sleeve, as can be seen in examples in the Tekniska Museet in Stockholm. Another description of the device can be found in [Ber1863].

11. Stabrowsky's Planimeter, Prussia, 1835

Actuarius Stabrowsky in Czerniejewo was granted a patent for a planimeter, for the term of 8 years from 26 August, 1835; described as a mathematical instrument for determining the area of plane figures [NN1835-1]. The patent had been granted in the same year (1835) the note got published [Sta1835p]. We could not find any further information on this planimeter.

12. Lalanne's Arithmoplanimètre, France, 1839

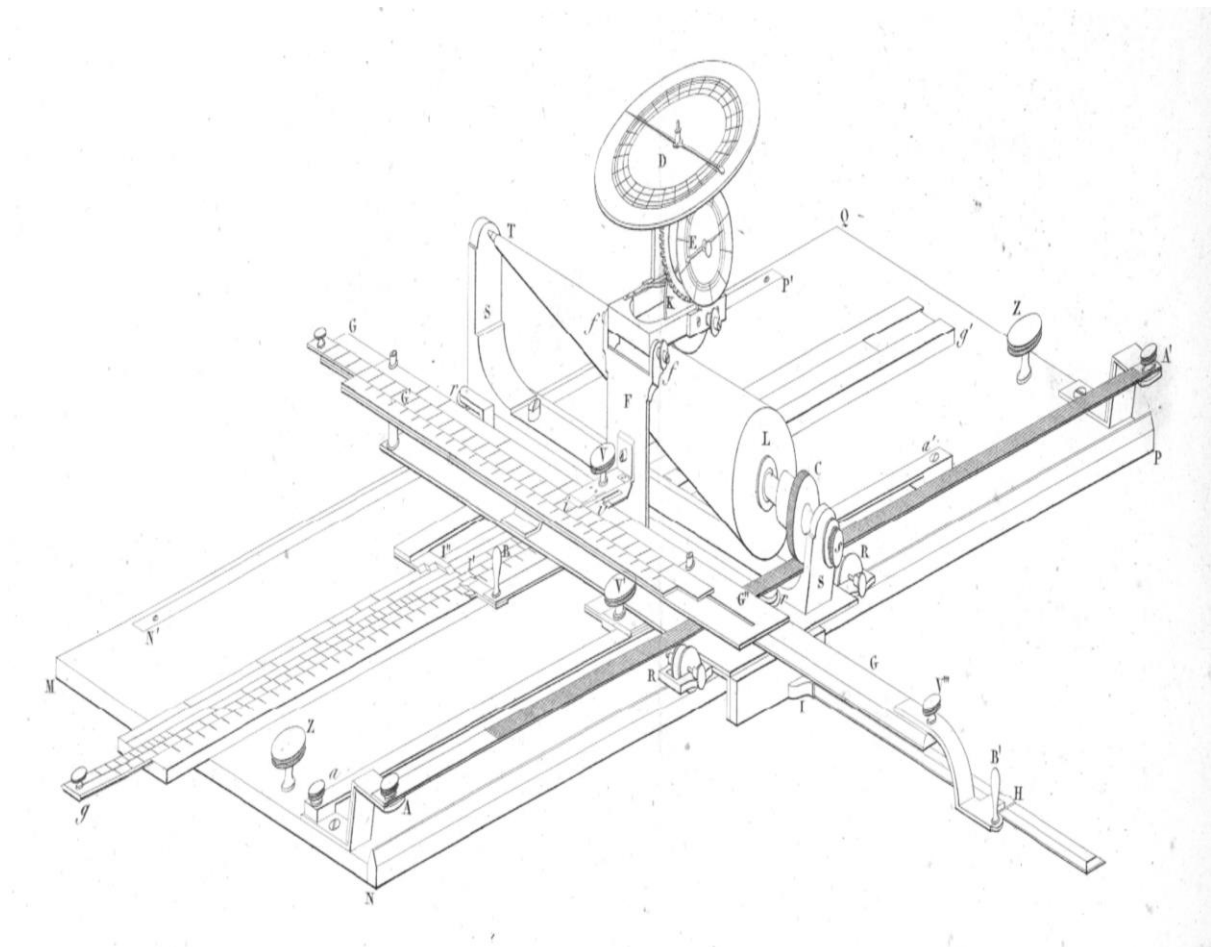


Figure 17: Lalanne's Arithmo-Planimeter.

Léon Louis Lalanne; real surname: Chrétien-Lalanne; (3 July 1811, Paris – 12 March 1892, Paris) was a French engineer and politician [Wik2023-5], [NN1892-6]. In 1839, Lalanne presented his Balance Arithmétique to the Academy, which was designed to calculate the distances required to move the cut and fill earthwork volumes needed to build roads, canals and railways [Lal1839-2]. The top of the beam was divided into 150 squares, each two millimetres wide; weights were placed in these squares: distances were measured to the nearest 150th of a meter, up to 600 meters. The earthwork volumes were represented by the weights; since one cubic meter corresponded to five milligrams, the approximation of one of these units could be used to work on a total of 20,000 cubic meters [Lal1876][Lal1839-1]. Lalanne, who wanted to make the calculations for which he had designed his Balance Arithmétique easier, naturally came up with the idea of the cone; but once he had heard of Ernst's planimeter, he was quick to recognize all the advantages it offered for shortening the long

and tedious calculations of earthwork volumes. In December 1839, shortly after the introduction of his Balance Arithmétique, Lalanne presented his Arithmo-planimeter to the Academy [Lal1840], [NN1840-1].

By adding two slide rules, one parallel and the other perpendicular to the direction in which the counter was set, he modified the existing planimeter by Ernst (1833) into a new instrument capable of quickly performing the most complicated operations. Instead of the simple products of two factors, it gave with an accuracy approaching 0.001, the products of any number of whole or fractional factors, whole and fractional powers, and even irrational powers of any degree as well as working with trigonometric functions. There was no need to draw auxiliary lines or measure the length of each line. Instead, each point of the figure whose area was to be determined had to be set twice by a double movement, i.e. four times in general. This seemed to be just as time-consuming as simply measuring the coordinates or dividing the figure into triangles, so there was no clear advantage over these methods other than ease of operation. The instrument was also large and complicated, susceptible to damage in transit, and expensive, all of which may have contributed to its limited success [JacPet1848].

13. The Computing Scale, England, 1839

The computing scale was in general use in England throughout the 19th and early 20th centuries. Robert Mason claimed to be its inventor when, in a letter to the editor of *The Surveyor, Engineer and Architect* in 1841, he stated that he had sent a description of a “scaling instrument” to the Tithe Commission in 1839 [Bru1840-2] & [Mas1841]. In 1836, the British Parliament had passed the Tithe Commutation Act, which was designed to replace the old system of paying tithes in kind with monetary payments. This led to the creation of the Tithe Commission, and the production of the tithe maps needed for the valuation process required by the Act [Wik2023-4]

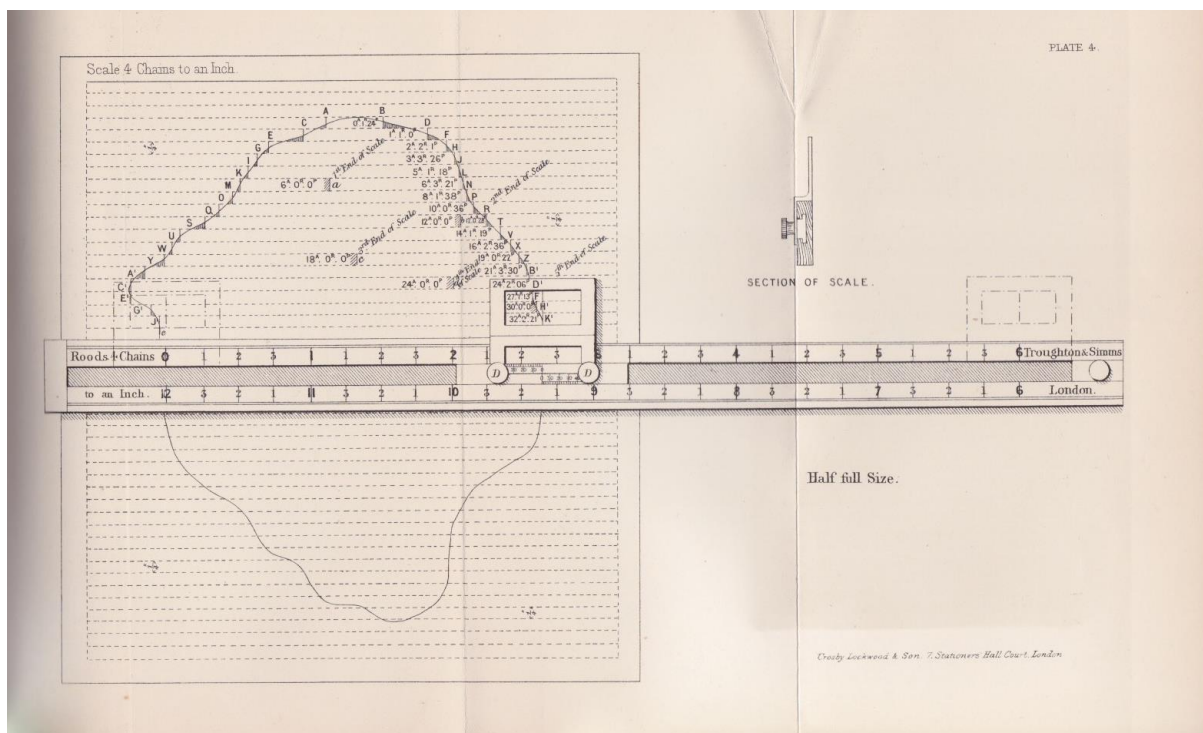


Figure 18: Computing Scale taken from [Usi1889-1], 5th edition from 1898.

In 1840, Peter Bruff, in his textbook on engineering fieldwork, described the instrument, which was “...now in the course of adoption by all surveyors, and at present exclusively employed at the Tithes Commission Office, presents the greatest facility in performing computations, without in the least damaging the plans, by equalizing boundaries, &c., as by all previous contrivances.” [Bru1840-1]. The instrument consisted of a scale in boxwood, available in any required length, with a moveable cursor in an undercut groove. In the method described, a tracing paper with parallel lines is placed over the enclosure to be calculated. The length of

each successive strip is added to the rule and the accumulated length of all the strips is read off the scale. This length multiplied by the common strip width gives the area sought.

Now called the computing scale, it was again described in detail in Charles Bourns' surveying textbooks in 1843 [Bou1843] as well as [Bou1845] and was subsequently covered in numerous land surveying textbooks by various authors. Several versions were developed and some were patented. Around 1867, Stanley patented an improved version of the computing scale with interchangeable scales, which allowed it to be used on maps of different scales [NN1867],[Gre2023-2], [Wil2022]. At about the same time the land-surveyor Henry S. Merrett patented an improved computing scale having 3 scales and described it in his textbook on land-surveying [Mer1863]. A later version of Merrett's improved scale with 4 different scales appeared in an article in The Builder in 1887 [NN1887-2].

The fact that the computing scale was listed in the catalogues of many British manufacturers and retailers of mathematical instruments was testimony to its popularity in England. In [Hae2011] a version from 1878 manufactured by Stanley is shown.

14. John Miller's Pantoscale, 1841

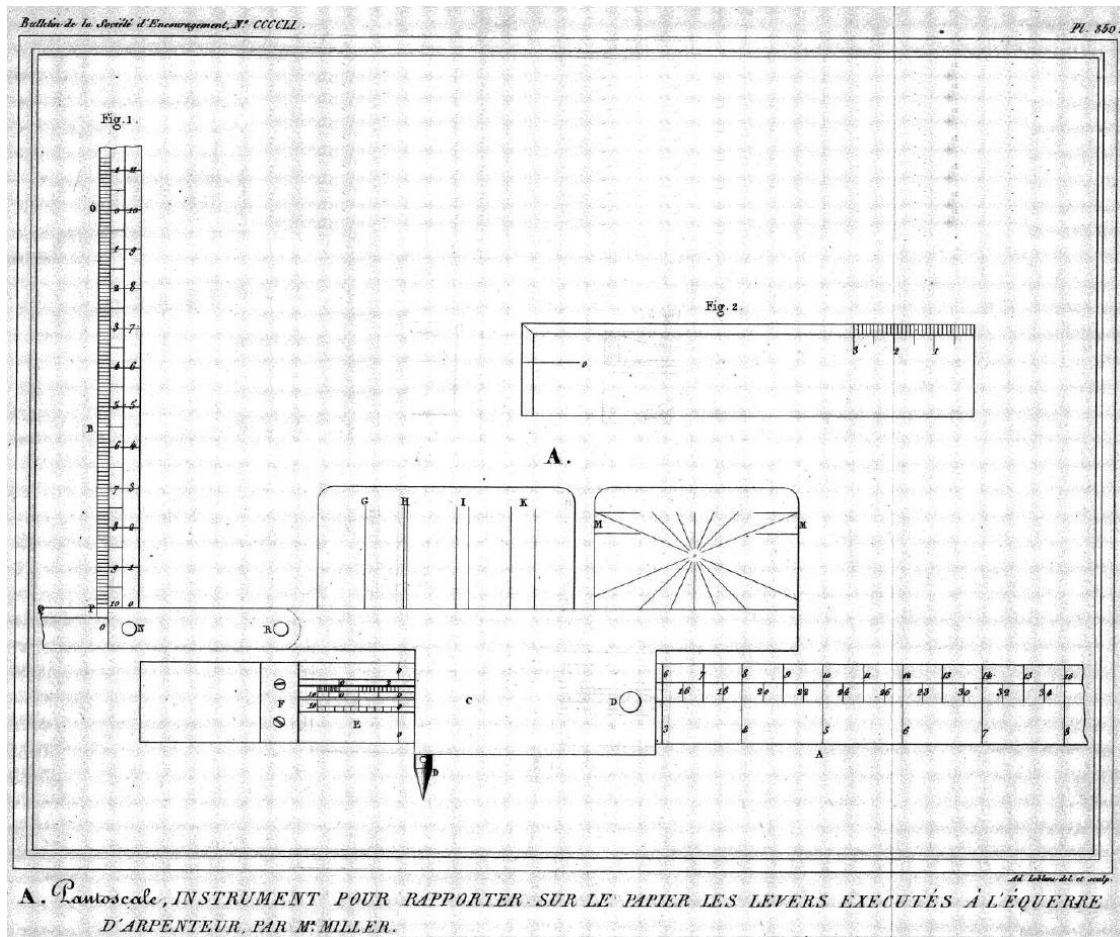


Figure 19: John Miller's Pantoscale [Mil1842].

In 1841 John Miller invented the Pantoscale [Mil1842]. The device consists of a 65 cm long horizontal rule (lower part of Figure 19), a 20 cm long vertical rule (left part of Figure 19), a carriage C, a compass D attached to the carriage, a steering rule E, a stopping point F corresponding to the zero position of the horizontal rule. In addition, two glass reticles, one with parallel lines (GHIK) and the other with parallel horizontal lines (MM) and fanned lines. The instrument was also supplied with an auxiliary brass rule (Fig. 2. In Figure 19).

The Pantoscale was clearly designed as a multi-purpose instrument, and Miller summarised the capabilities of the instrument as follows:

1. to plot the length of any line very accurately;
2. to draw parallel lines;
3. to plot angles with a radius of 20 centimetres using sine and cosine tables;
4. to construct triangles more quickly and accurately than with a compass and rule;
5. to plot coordinates and curves of any radius;
6. to copy, reduce or enlarge plans;
7. to determine the area of irregular polygons;
8. to divide any area into equal or unequal parts.

In this sense, Miller's Pantoscale might not be considered a planimeter as such, but rather an all-in-one instrument combining the capabilities of a planimeter, a coordinatograph, a section-liner, a protractor and a pantograph. Miller later renamed it the Polygraphe. The instrument could be purchased for 40 Francs from M. Miller, Rue du Temple, 69, and from Lerebours, Place du Pont-Neuf, 13.

15. Roulette de Dupuit, France, 1844 & 1878

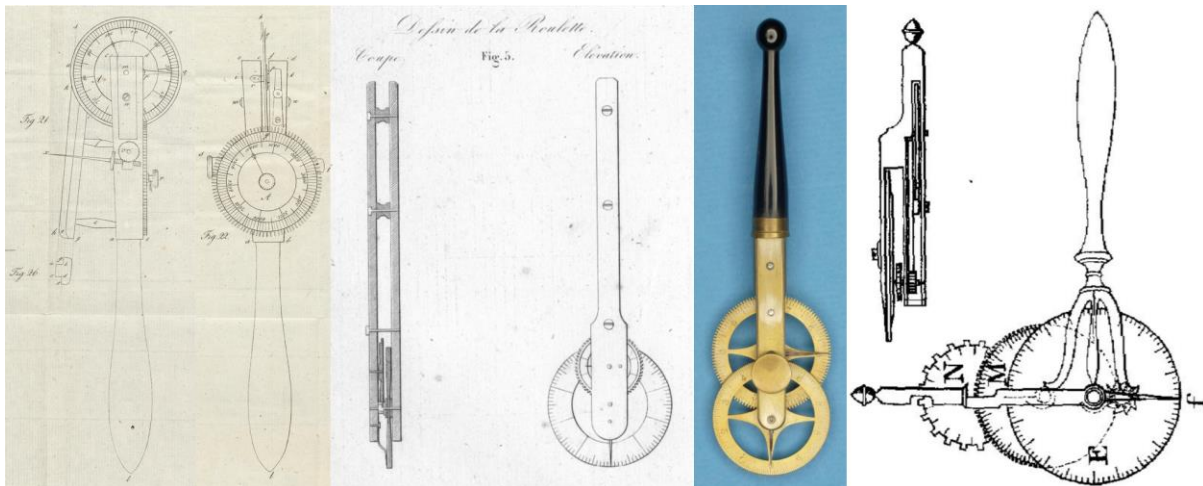


Figure 20: Reinventing the wheel: four strip measuring devices from three different countries and four different time periods. From left to right: Oldenburg [Pie1825] (see also [Zer2021-4]), Roulette de Dupuit [Dup1844] and improved version from 1875 (photo) and Erichsen [Eri1884] (drawings on the right, described in section 31 below).

The French civil engineer and economist Jules Dupuit (1804-1866), wishing to relieve the engineers who were charged with the long and tedious task of calculating earthwork volumes involved in public works, devised a handy portable device which came to be known as the Roulette de Dupuit. It was used with the help of a tracing paper placed on the cross-section whose area was to be calculated, on which a series of equidistant parallel lines were drawn. Dupuit published an essay on his invention in the *Annales des Ponts et Chaussées* in 1844 entitled “On an instrument intended for the rapid calculation of all surfaces and in particular those of cut and fill” [Dup1844].

The Roulette de Dupuit consisted of two wheels, the smaller of which had a circumference of 100 millimeters; an index with a pointer was attached to the handle, the axis of which carried a pinion that drove a large wheel with ten times more teeth than the pinion. The circumference of the large wheel was divided into ten equal parts, and each division, indicated by an index, corresponded to one revolution of the small wheel, or 100 millimeters. The large wheel had a maximum range of 1000 millimeters, which, according to Dupuit, was sufficient for all ordinary cases [Dup1844]. The device was also known in Germany [NN1845-2].

In 1875, an improved version of the Roulette de Dupuit appeared in a textbook on land-surveying “Arpentage Levé des Plans et Nivellement” by F. J.-O. P. [NN1875-1]. The Roulette de Dupuit was still in use by civil engineers well into the 20th century [Mor1910].

16. Hennon's Polygonometre, France, 1846

The inventor of the polygonometre was V. Hennon, géomètre forestier from the town of Cosne, a commune in the Nièvre department in central France. He was the author of a textbook on forest surveying entitled “Géodésie Pratique des Forêts” published in 1846, with a second edition published in 1851 [Hen1846]. The polygonometer is described in both editions of this textbook. On pages 75-76 Hennon states the following: “...Already several geometers had occupied themselves with this rather important object with more or less success; but as of their methods were limited to the conversion of a polygon into an equivalent triangle, and that they could not give the surface immediately without calculation or dividers, I wanted to make up for this defect by seeking to these two advantages in a single instrument, and I believe I have succeeded in doing so by the invention of the polygonometre as shown in fig. 57. This instrument has the double advantage of converting a polygon of 1 to 25 hectares (scale of 1 to 2,500) into an equivalent triangle and to give the surface to the whole in less than a minute, without calculation nor compass, and without drawing on the paper any other line than a single, light pencil line on the paper. It is as simple and as easy in its use, as it is exact and expeditious in its results.”

The Polygonometer, in case, was available from Hennon in Cosne at the price of 60 Fr.

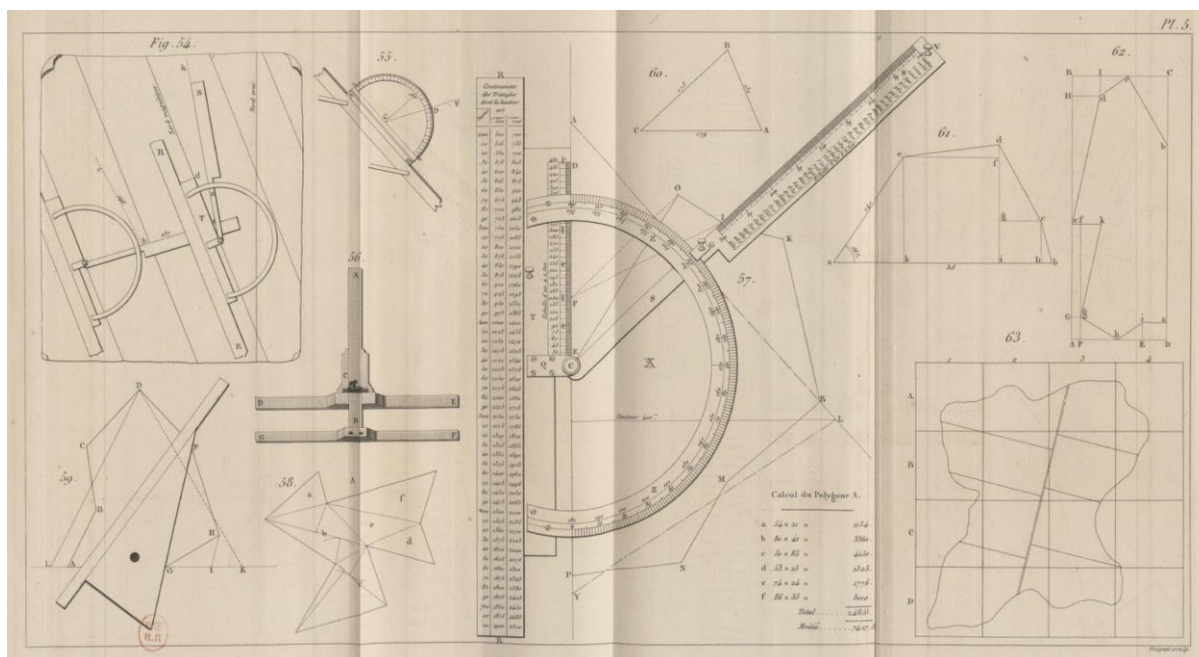


Figure 21: Hennon's Polygonometre taken from [Hen1846].

17. Godfray's Agrometers, England, 1848

Hugh Godfray (1827-1877) was a navigation mathematician and astronomer. During 1846 and 1847 he surveyed the island for his Map of Jersey, published in 1849 [Jer2023]. He frequently contributed articles to the *Mathematician* and other scientific journals, and so became known to scientists in England. He described his rectangular and circular Agrometers in *Mechanics Magazine* [God1848]. Both instruments work generally along the same lines. The tracer is connected to rods that drive a gear wheel, which is attached to a second wheel that runs on a disc, on which the result can be read. Godfrey proposed that the rims of the disc should be wide enough to accommodate several rows of divisions corresponding to the different scales at which plans are usually drawn. Although cleverly conceived, Godfrey designed both instruments as a means of exploring mechanical and mathematical solutions, but no model of either was ever actually built.

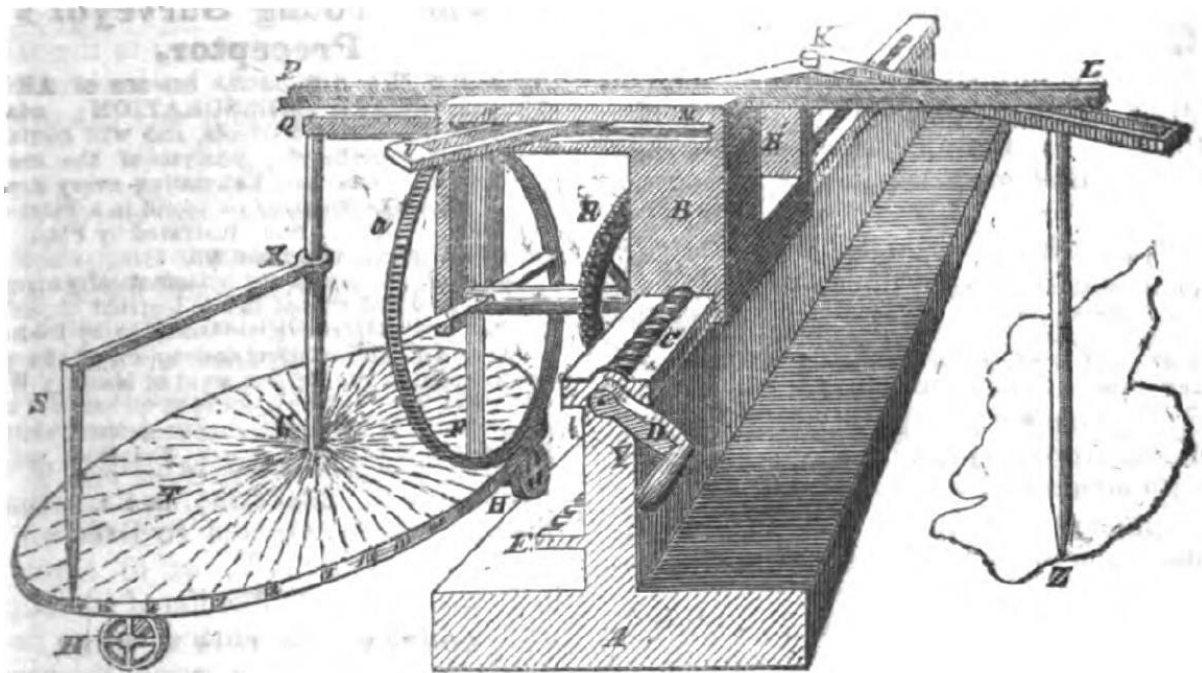


Figure 22: Godfray's Agrometer from [God1848].

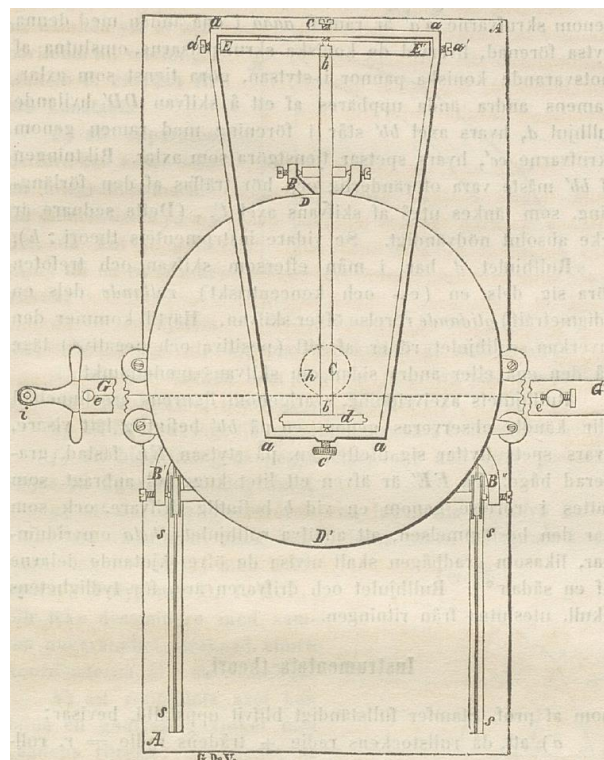


Figure 23: Källström's patent planimeter from [Kal1852]
(Courtesy of Kungliga Biblioteket, Sweden)

18. Källström's Planimeter, Sweden, 1852

Forest manager Wilhelm Emanuel Källström (1811-1868) obtained a patent (Swedish pat. no. X930, 15.03.1852)[[Kal1852p]] for five years for the manufacture of a version of the Wetli planimeter, and described in

his “Om Patent Planimetern” published in 1852 [Kal1852]. Källström is also known for his “Produkt-och reciprok-tabeller”, published in 1849 [Kal1849]. The Swedish Patent Archives provide a basic listing of X930. However, the original patent document with a description has not been made available online.

19. Gierer’s Planimeter, Germany, 1854

Based on an article by Bauernfeind on his “Ringmesser”, Johann Andreas Gierer decided to design a similar device [Gie1854]. Since Gierer’s invention has already been described in Stefan Drechsler’s article [Dre2010] we only refer to this integrating planimeter without giving further details.

20. Burman’s Planimeter, Sweden, 1852 & 1856

Lars Abraham Burman (1805-1876) (page 41 in [Eks1903]) land-surveyor, patented in 1852 an improved planimeter (Swedish pat. X983, 26.11.1852)[Bur1852p] based on the Wetli planimeter. Burman patented a planimeter again in 1856 together with court clerk C. Cavallin (Swedish pat. X1258, 23.09.1856)[BurCav1856p] for a term of 8 years [NN1856-2]. The Swedish Patent Archives provide a basic listing of X983 and X1258. However, the original patent documents with a description has not been made available online. We have no further information or drawing on this planimeter.

21. Decher’s Planimeter, 1855

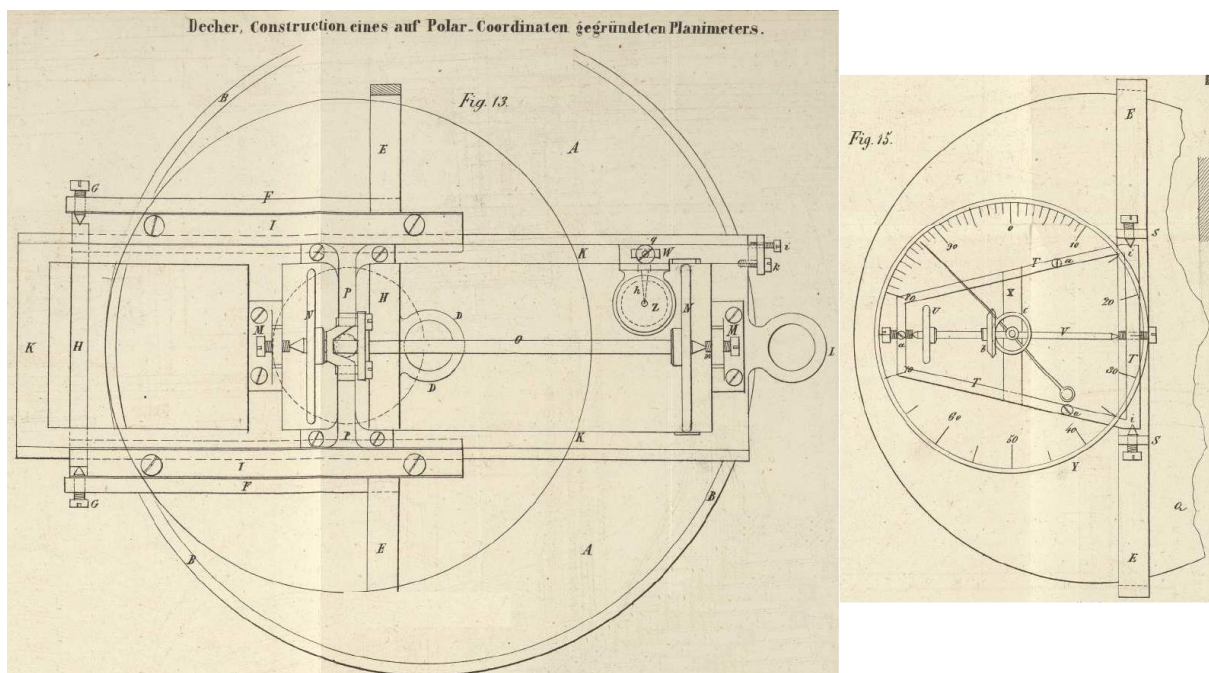


Figure 24: Decher’s proposal for a planimeter based on polar coordinates [Dec1855].

In his article [Dec1855], G. Decher, professor at the University of Augsburg, criticised both Wetli’s construction and Hansen’s improved version of Wetli’s planimeter in relation to Wetli’s planimeter described in Stampfer’s articles [Sta1850-1], [Sta1850-2]. Without giving a reason, Decher rejected instruments in which all movements took place around a fixed vertical hub (e.g. a fixed pole of a polar planimeter). Decher proposed a different approach as shown in Figure 24. Although Decher was convinced of his simpler design, at the time of publication [Dec1855], five years after Wetli’s invention, he still had not found a manufacturer for his device. In his article he described the theory of his planimeter and tried again to get his approach accepted.

In [Dec1856-1] Decher presented a modification of Bouniakovsky’s planimeter (Figure 26) by avoiding two arms of the rhombus-shaped planimeter. But once again, Decher’s idea was not accepted. Without providing any concrete evidence, he continued his persistent criticism of the Amsler-type polar planimeters. He remained sceptical about the idea that a combination of a sliding measuring wheel and a rolling measuring wheel could give a reliable result, stating that (translated from German) “the principle cannot be recommended,

as it can never guarantee the necessary reliability in practice, even if it is theoretically correct". Ultimately, the imminent success of the Amsler Planimeter was to prove Decher wrong.

22. Stadler's Hyperbole Planimeter, Austria, 1855

There is a brief mention of Stadler's hyperbole planimeter in [Tou2022], but we will endeavour to describe it in more detail here. It is the so-called hyperbole planimeter which had been manufactured in 1855 and had been described in [Sta1858]. The inventor Josef Stadler was born in Nussdorf near Vienna in 1816 and died in Eisenerz in 1871 [NN1872-2]. He was mining councilor and director of the imperial mining and smelting works in Eisenerz, an old mining town in the Austrian province of Styria. In the mid-1850s, Stadler became aware of the planimeters of Gonnella and Oppikofer, but as he was unable to see examples of the actual instruments, he set out to invent one of his own, which he succeeded in doing [NN1883-1]. Stadler commissioned the mining official Georg Smollin, who had a reputation in Eisenerz (Austria) for being mechanically gifted, to construct the planimeter according to Stadler's specifications. Over the course of 1855, Smollin built three Stadler Hyperbole planimeters in two versions, one with linkages and the other with a carriage [Hin1883]. Stadler mentioned that the device had an accuracy of typically below $\frac{1}{2}$ percent.

Stadler's planimeter belongs to the strip measurement devices and is similar to Oppikofer's and Hermann's in that the area is determined by a rotating roller. But the idea is different from Oppikofer's in that the rotation of the roller is proportional to the width of the measured stripes of the area, i.e. that the multiplication of the strip length times strip width is mechanically done by choosing a fitting circumference of the roller. E.g. a roller circumference of one inch is used and the width of the measured strip is one inch, then the number of revolutions of the roller is identical to the area of the strip in square inches. If the strip width is a half inch, then a roller circumference of two inch should be used, such that the number of roller revolutions refers to the area in square inches again. To combine the different circumferences on one roller results in a conical roller, like the ones in Oppikofer's and Hermann's device. At this point Stadler's construction differs by using a funnel-shaped roller. The reason for his design is that the funnel-shaped roller cannot directly roll on the boundary of the figure to be measured. Stadler solved this problem by introducing a rotating tangential rod rolling on the funnel-shaped roller (see Fig. 6 in Figure 25). The tangential rod is connected to the tracer of the planimeter and is driven by the movement of this tracer. The surface of revolution of the funnel, whose meridian is an equilateral hyperbola and whose axis of rotation is an asymptote [Sta1858]. Hence the Hyperbole planimeter's name.

Stadler's hyperbole planimeter was only intended for use at the mining works and remained relatively unknown. It was by chance that Hinterhölzl discovered the existence of Stadler's planimeters and had 2 variants (one with a tracer and another with a carriage) of them exhibited at the Ausstellung culturhistorischer Gegenstände in Steiermark in 1883 ([Cen1883] page 126). In the same year Hinterhölzl exhibited the Stadler planimeter at a meeting of the Polytechnischer Club in Graz [NN1883-1], [Hin1883]. Hinterhölzl noted: *"...Apart from the fragments of a third one, only these two exist, which are currently in the Geodesy Department at the Imperial and Royal Technical University in Graz as a landmark of scientific endeavour in Styria. They were not created out of necessity, but rather the product of study out of interest, and Stadler himself said, "I later found that my planimeters are no simpler than those that already exist."* [Hin1883], [Dyc1892] page 184.

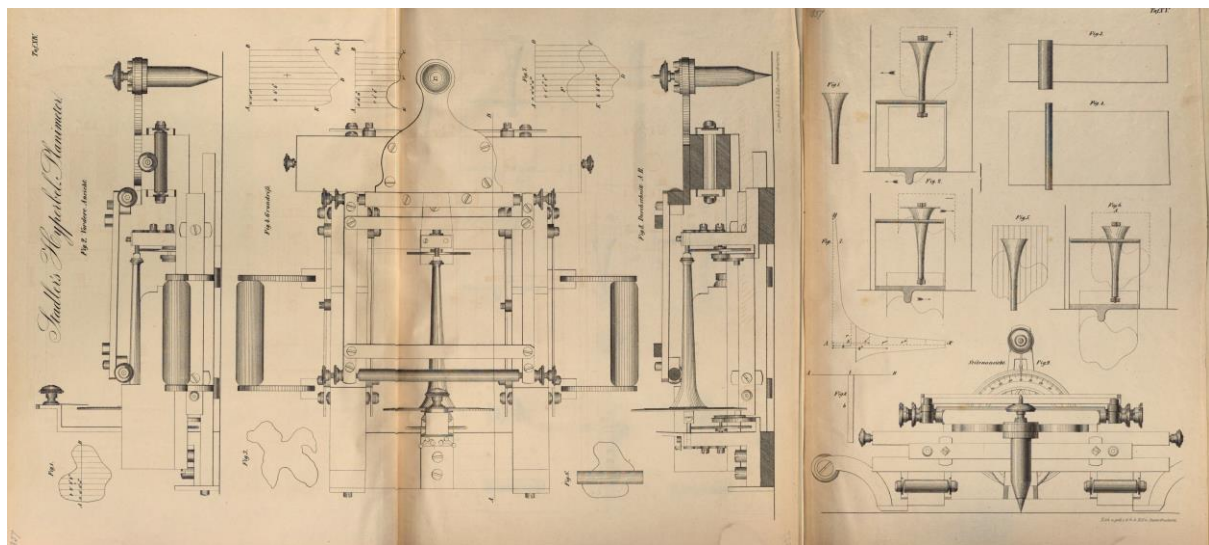


Figure 25: Stadler's Hyperbole planimeter from 1855 ([Sta1858]).

23. Buniakovsky's (Bunyakovsky) Planimeter, Russia, 1855

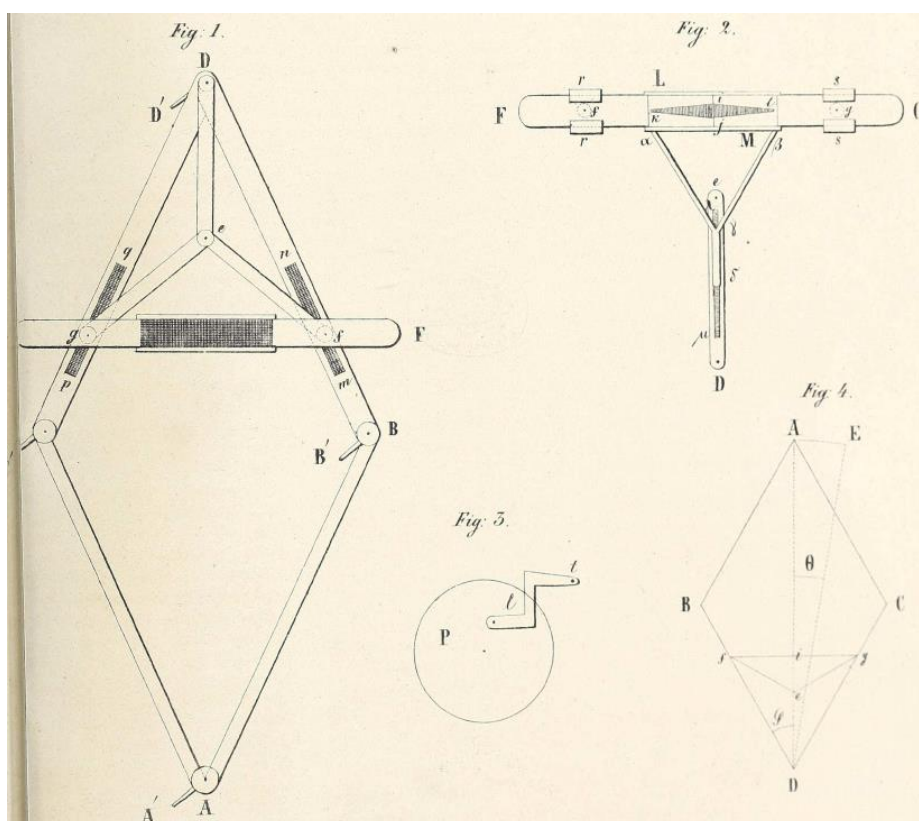


Figure 26: Buniakovsky's planimeter from [Bou1855].

In parallel with the development of various strip measuring planimeters, integrating polar planimeters were also being invented. An early version is the polar planimeter of the Russian mathematician Viktor Buniakovsky (1804 - 1889) [Wik2023-6].

The planimeter and its theory are described in French [Bou1855] and in German [Sau1856]. Its operating principle (see Figure 26, see Figure 1) is based on its deformable rhombus (see Figure 26). During the measurement the corner D is fixed on the paper carrying the area to be measured. The perimeter is traced with cursor A. During this movement the angles of the rhombus change. The measurement device itself is integrated in an additional bar GF mounted on top of the rhombus. This bar is guided in points g and f sliding in the slots pq and mn respectively. Figure 26 shows the bottom view of the measuring bar. In LM the measurement wheel is shown, which stands orthogonal to the paper. Based on the movement of the tracer A, there is a mechanically enforced movement of the measurement bar and the corresponding measurement wheel. Depending of the direction of the movement, the wheel performs a combined rolling and sliding motion. When the operator has finished tracing the entire perimeter, the size of the area can be read off the measuring wheel using a Vernier.

24. Dasnoy's Calculateur-Rapporteur, Belgium, 1856

Our journey now takes us to the west of Europe. Jean Baptiste Dasnoy was born 10 July 1918 in Massul, Belgium; and died 9 November 1886 in Namur [NN1900-1]. He held the position of Contrôleur du Cadastre in Arlon. Dasnoy was familiar with Gelinski's Calculateur and invented an improved version. In 1856 he published a treatise on its usage "Instruction sur l'usage de Calculateur-Rapporteur de Gelinsky, perfectionné par Dasnoy, Geometre du Cadastre..." [Das1856].

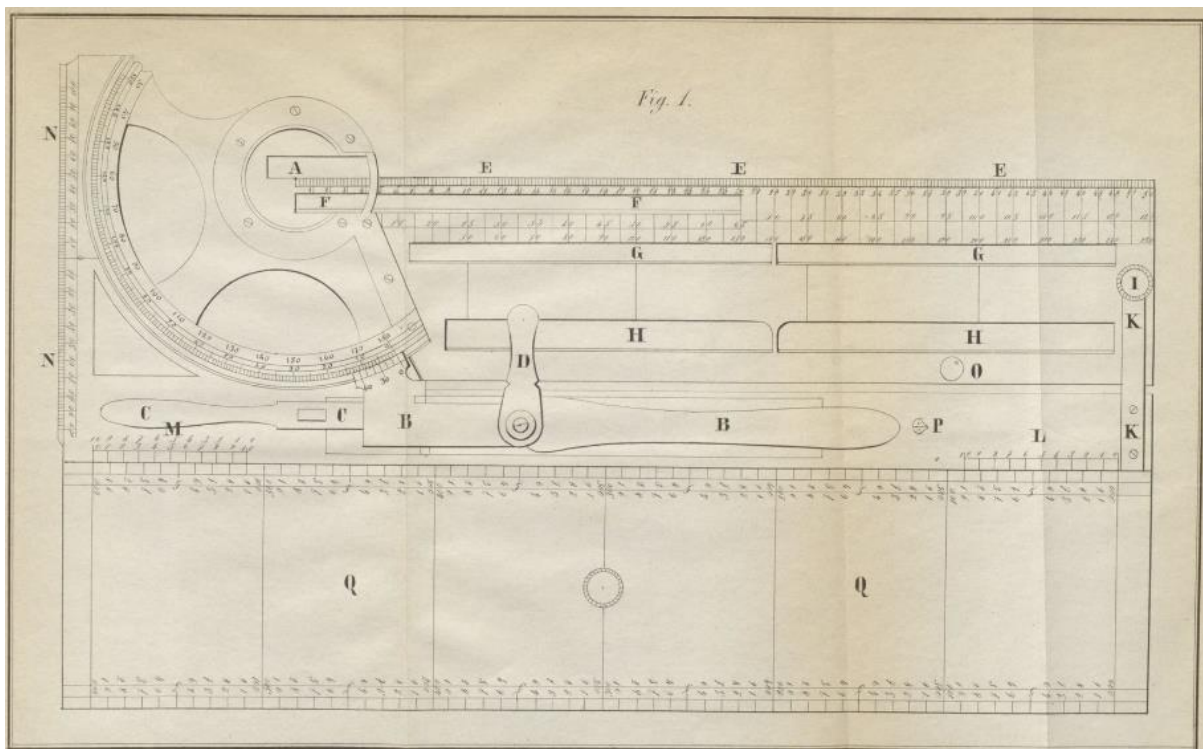


Figure 27: Dasnoy's Calculateur-Rapporteur from [Das1856].

Dasnoy detailed his improvements to Gelinski's Calculateur as follows:

1. replaced the glass wing with a copper plate pierced with windows, making the construction of the instrument much more precise, less expensive and more solid.
2. replaced the hinge with a circle that did not allow any swinging, and added a spring lever that prevented the wing from closing by sliding, while allowing free and frictionless rotation.
3. added a detachable extension which makes it possible to measure larger parcels.
4. made a groove to prevent the instrument from moving away from the ruler.
5. placed the centre line of the limb at the edge of the wing, making it possible to draw reference lines when transforming the figures and giving a far greater precision.
6. modified the instrument to be able to calculate angles, perpendiculars and distances, and to divide lines.

As already mentioned in the section on Zaręba's planimeter, Gangloff's device is based on the same polygon transformation to a triangle of same size. Gangloff, a forestry officer, provided his device for a demonstration in front of the "mährisch-schlesischen Forstsection" [NN1856-4]. An original description of the device is given by Gangloff [Gan1857] and Grabowski [Gra1908-2]. Even though the corresponding devices (Zaręba Figure 7 and Gangloff look quite different, the devices are used in a similar way. Gangloff's device uses a glass plate C (of size 42 x 26 cm) on which the rotatable arm B (with a crosshair) is mounted. Since the rotatable arm is mounted on the glass plate, it cannot unintentionally rotate during the parallel shift of the glass plate. In fact, Grabowski noted in his article [Gra1908-2] that the positioning of the crosshairs over the polygon to be measured was less accurate compared to Zaręba's planimeter. The reason for the inaccuracy is that the reticle is above the glass plate and therefore not in the same plane as the polygon to be measured.

26. **The Transformateur, the Netherlands, 1867.**

84

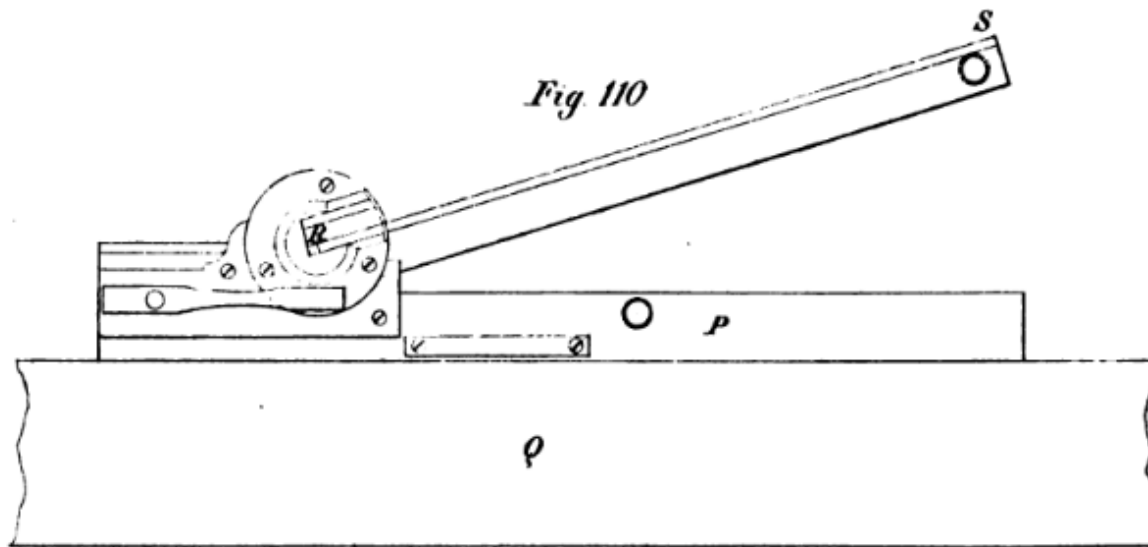


Figure 29: The Transformateur, widely used at the Netherlands, taken from [Sch1891].

27. Hofmann's Instrument, Germany, 1874.

With reference to Dasnoy's Calculateur-Rapporteur described earlier, Hofmann's device must also be mentioned. Karl Hofmann was born on 23 September 1811 in Bettenbrunn, Germany, and died on 1 September 1884 in Karlsruhe, Baden, Germany [NN1884-6]. He held the position of Vermessungsinspektor. In 1874, Hofmann's instrument was described in an article by the geometer M. Doll in the Zeitschrift für Vermessungswesen [Dol1874]. In this article, Doll attributed the invention solely to Hofmann. Doll may not have been aware of similar instruments by Gelinski or Dasnoy, as he makes no mention of either. In response [Hof1874] to Doll's article, Hofmann confirmed that he was not the inventor of the instrument, but pointed out that he had made only a few modifications to Dasnoy's instrument, based on suggestions directly from Dasnoy himself.

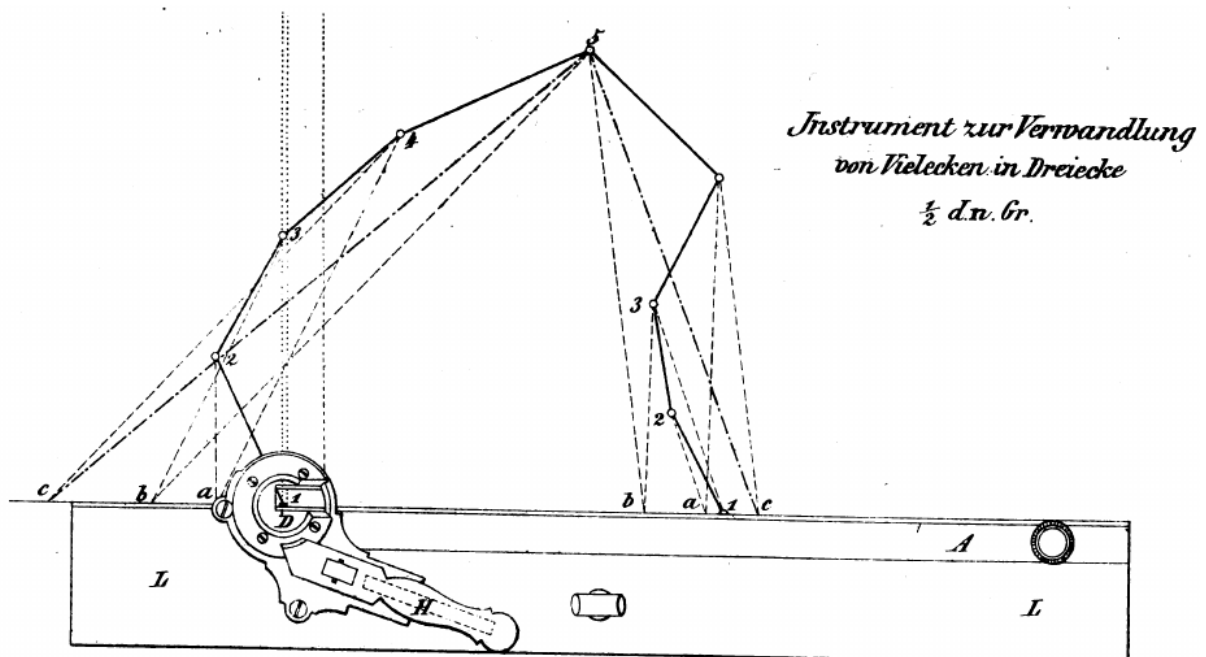


Figure 30: Hofmann's device taken from [Dol1874].

Hofmann's instrument was available from the instrument maker Carl Sickler in Karlsruhe and is known to have been listed in his catalogue. Jordan (*Handbuch der Vermessungskunde*, 1897, 5th edition of [Jor1890]) commented on page 86 that the instrument had been in use in Baden for many decades.

28. Ljungström's Cirkelplanimeter, Sweden, 1870

Jonas Patrik Ljungström (1827-1898) land surveyor, inventor and cartographer [Wik2023-2]. Ljungström had a workshop for the manufacture of surveying instruments in Stockholm. Upon his death in Stockholm in 1898, he was succeeded by his son instrument maker Axel Ljungström (1864-1919). Axel Ljungströms Fabriks AB was acquired by Instrumentfabriks AB Lyth in 1917 [Wik2023-3].

The exact year of the invention of Ljungström's Cirkelplanimeter [Lju1870p] is unknown and it is not listed in the archives of the Swedish Historical Patents (1746-1945). Its first mention is in an article posted in the *Aftonbladet* of 1870 [NN1870]. Here the Cirkelplanimeter is described as "... of great practical value to surveyors, engineers, shipbuilders and other persons who deal with practical geometry, and the present construction of it has been recognized as ingenious and particularly useful." The article also mentions that Ljungström had previously constructed an improved version of the Wetli planimeter, which won a prize at the Industrial Exhibition in Stockholm in 1866 and was subsequently purchased by the state.

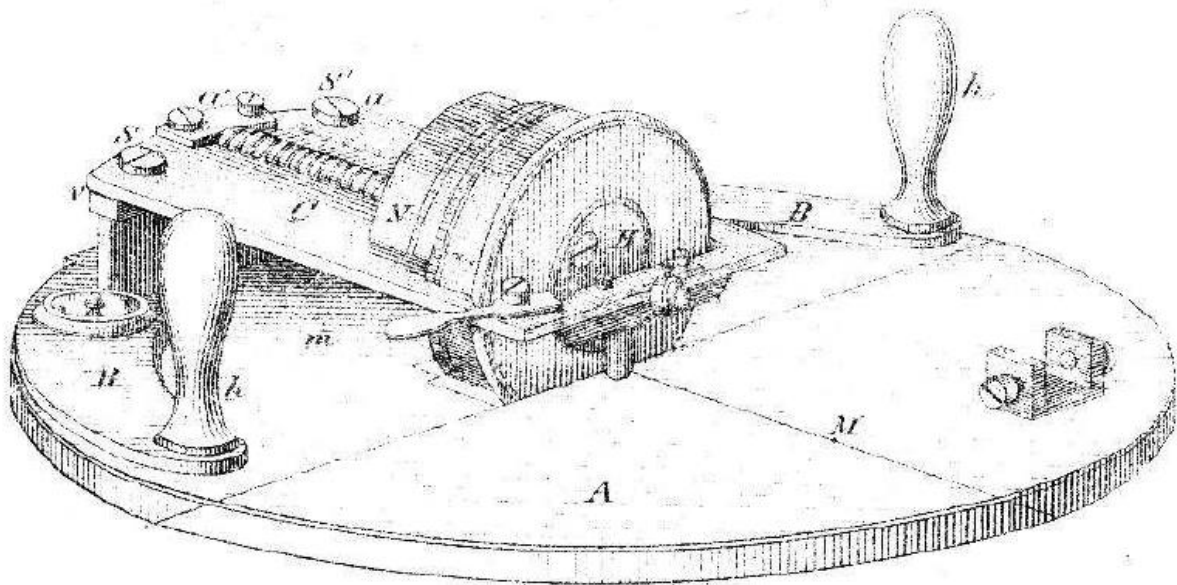


Figure 31: Ljungström's Cirkelplanimeter from [And1876] (see page 241).

Among Ljungström's entries at the International Exhibition of 1876 in Philadelphia was his Cirkelplanimeter. In the section on Scientific and Philosophical Instruments and Methods in the Swedish Catalogue of the exhibition the following description is given: "*For the purpose of measuring geometrical figures, J. P. Ljungström, the Engineer, has constructed a so-called circular planimeter, consisting in its most simple form of a circular plate of glass, through the centre of which passes a vertical steel wheel, that ought to be placed at right angles to a radius which is etched in the pane and marked with a point. When in use, the plate of glass is brought along a rectilinear support (a ruler) so that said point accurately follows the border line of the figure once round. The reading is taken from the wheel which then is turned by friction towards the paper. The instrument measures figures of a width, equal to that of the whole ruler. If very large figures are to be measured (e. g. an ordinary map at once) the instrument is changed, first by a so called "couple" into a kind of polar planimeter. The "couple" consists of a metal ring, which goes round the pane of glass, together with two legs, one of which is attached to the ring, and the other to the plate. The former moves about a fixed point, while a point marked on the other leg, is moved along the border-line of the figure. The reading off is in this ease also done from the steel wheel.* [Sid1876]

29. Mechanised Harp Planimeters, Sweden, 1861 - ~1900



Figure 32: Liedbeck's mechanized harp planimeter, 1868, serial number 129. (Photo: Stockholms Auktionsverk)

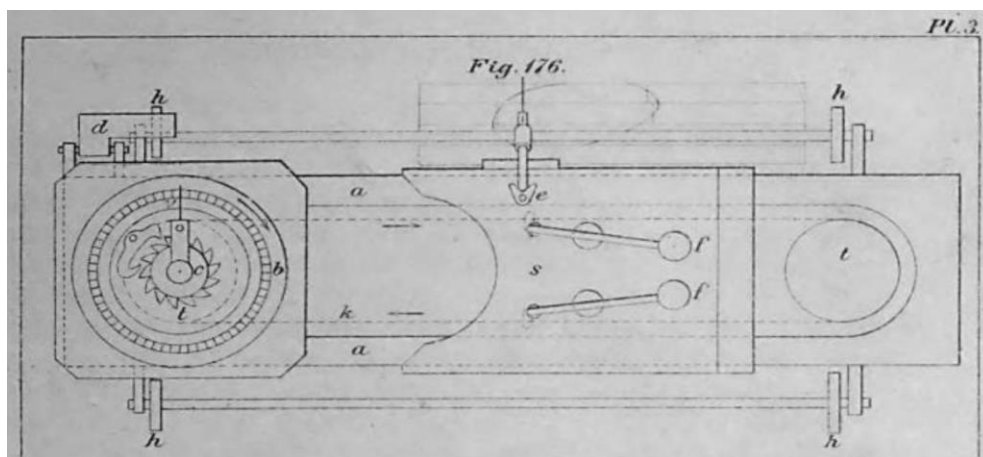


Figure 33: Liedbeck's planimeter taken from [Ant1878].

Again we have to break our chronological order, because after 1861 a separate development and a completely different design spread in Sweden, but not beyond. Here we have to make some corrections. In section 11 of [Zer2021-4], the mechanical harp planimeter described was incorrectly attributed to Ludvig Köhler. Based on David Green's feedback, the planimeter was most likely made or sold by Alex Ljungström. The very special construction of this type of planimeter seems to be limited to the Swedish area. The design of a chain-driven carriage moving over a glass plate with an engraved grid, resting on paper with the area to be measured, is quite unique. So far, 15 planimeters based on this principle have been found in existence. The corresponding list is given in [Table 1]. Interestingly, at least four (Swedish) manufacturers have already been identified among these 16 instruments ([Gre2023-1]).

In [And1876] (page 225) Andersson describes Liedbeck's planimeter as follows (translated from Swedish): "Liedbeck's surface calculator is composed of the following main parts: a frame *a*, four roller wheels *h* - two and two attached to the same axle - by which the instrument can be moved parallel to itself closer to or further away from the user; an endless chain *k*, stretched around two pulleys *t*, the left of which is provided with table *e*, resting on the fixed table and movable about its center; a brake valve *d* trailing the roller wheel against one side and a disk movable along the frame and equipped with a handle, the runner *s*, which supports a diopter *e* and two keys *f*, by means of which the chain can be fastened to the runner alternately on one side or the other." Probably the design is based on Liedbeck's patent from 1861 [Lie1861p].



Figure 34: Mechanised harp planimeters from additional four different Swedish manufacturers. From left to right: Sköldin (according to the design by Liedbeck [And1876]), Ljungström, Köhler and Berg.

No.	Manufacturer	Remarks	Sources
1	N. F. Liedbeck	Serial number 129 (ca. 1868)	https://auctionet.com/de/2433374-oberflächenrechner-ca-1860-1870-cj-schild-nummeriert-129-holzrahmen-kufe-aus
2	O. J. Sköldin	Serial number 133 (ca. 1876)	https://www.uraone.com/PlanimeterVault/hair/Skoeldin133a.jpg
3	O. J. Sköldin	Medelpads fornminnesförening (Medelpad's antiquities association)	https://digitaltmuseum.se/0210211426900/arealutrakningsinstrument-i-lada-av-tra-planimeter
4	O. J. Sköldin	Serial number 184, collection Green	https://www.uraone.com/PlanimeterVault/hair/Skoeldin184a.jpg https://www.uraone.com/PlanimeterVault/hair/Skoeldin184b.jpg
5	Ljungström	Tekniska museet	https://digitaltmuseum.se/021026330841/rullplanimeter
6	Axel Ljungström	David Green's webpage. Manufacturing year 1898 or later.	https://www.uraone.com/PlanimeterVault/hair/Ljungstroems1a.jpg , https://www.uraone.com/PlanimeterVault/hair/Ljungstroems1b.jpg , https://www.uraone.com/PlanimeterVault/hair/Ljungstroems1c.jpg
7	Ljungström	David Green's webpage	https://www.uraone.com/PlanimeterVault/hair/Ljungstroems2a.jpg , https://www.uraone.com/PlanimeterVault/hair/Ljungstroems2b.jpg
8	Ljungström	Collection Zerfowski	[Zer2021-4]
9	Ljungström	Possibly manufactured by P. M. Sörensen	https://digitaltmuseum.se/021026330841/rullplanimeter

10	Anton Ludvig Köhler	David Green's webpage	https://www.uraone.com/PlanimeterVault/hair/KoehlerReddit2.jpg
11	Anton Ludvig Köhler	Collection Green	https://www.uraone.com/PlanimeterVault/hair/P1070230Koehler.jpg
12	Anton Ludvig Köhler	Tekniska museet	https://digitaltmuseum.se/021026349684/planimeter
13	Anton Ludvig Köhler	J. Vossepoel's webpage	[Vos2023]
14	Anton Ludvig Köhler	David Green's webpage	https://www.uraone.com/PlanimeterVault/hair/KoehlerMetropol1a.jpg , https://www.uraone.com/PlanimeterVault/hair/KoehlerMetropol1b.jpg
15	Anton Ludvig Köhler	Tekniska museet	https://digitaltmuseum.se/021026354318/planimeter
16	J. Berg	ca. 1899	https://mm-instruments.com/instruments/5a9217813834377cd4952ad5.html

Table 1: Known existing Swedish mechanised harp planimeters.

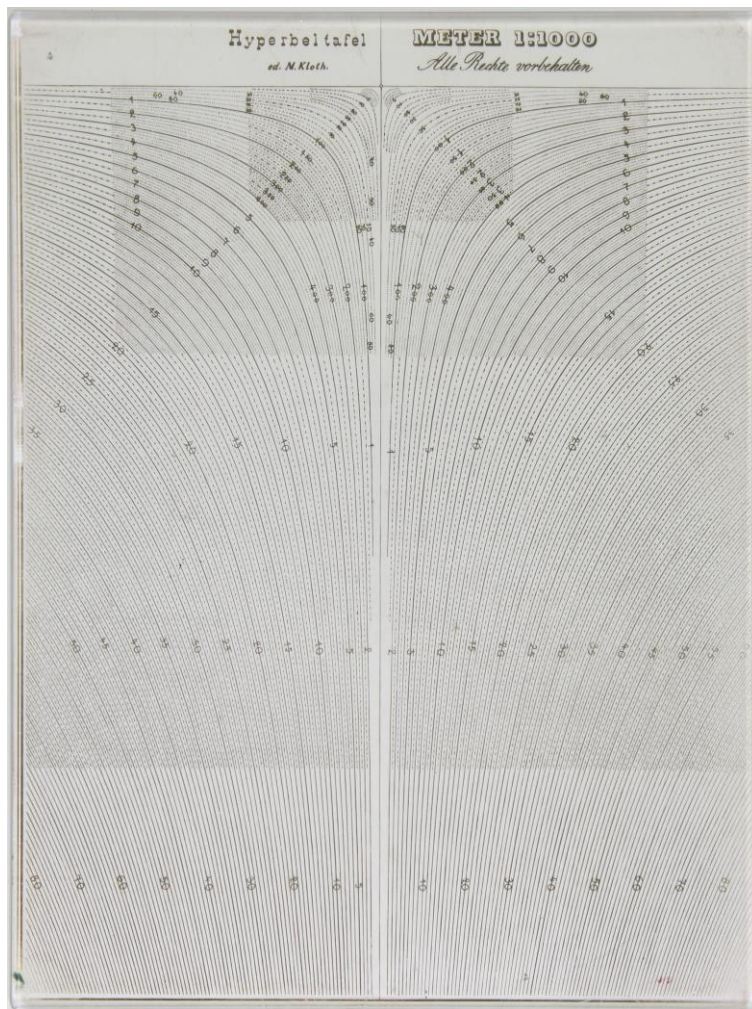


Figure 35: Kloth's Hyperbolic Plate (Source: [Cor2023], courtesy www.mathinstruments.ch).

With these Swedish harp planimeters an interesting technical detail came up. Since with one of the author's device the closed metal chain running around the big two wheels, we were looking for a corresponding spare part. Surprisingly, finding such a spare part is extremely difficult. Initially looking for a closed metal thread didn't succeed. Detailed photos of existing devices surprised us even more. It's really a 40 cm metal chain, with chain links of about 0.5 mm width and about 1.8 mm length. Therefore, a complete chain exists of around 220 links. So far, we did not find any person who can produce such a spare part for an affordable price.

As a summary on these Swedish mechanised harp planimeters, it is quite difficult to bring the devices into a timely sequence. For example it is known that Axel Ljungström took over his father's business in 1898 [Vos2023]. Therefore the fifth entry in Table 1 can be dated 1898 or later. But it's not clear whether it was Axel Ljungström who introduced the planimeters into the company's product range, or his father.

30. Kloth's Hyperbolic Plates, Germany, 1883

Although the use of hyperbolic tables to determine triangular areas had been introduced long before Max Kloth (see e.g. section 5 on Tenner), it is worth mentioning the invention of the so-called Kloth's hyperbolic plates. In 1883 Kloth applied for a German patent [Klo1883p] with the (translated) title "Transparent scales and calculation plates, and process for their manufacture". In the German surveying journal ("Zeitschrift für Vermessungswesen"), Wilski described Kloth's plates in his article on new calculating devices [Wil1892-2]. In addition to describing their use, Wilski rated the performance of Kloth's hyperbolic plates much higher than older devices using curves. But he also made some criticisms about the accuracy. In January 1893 Schelbach published another accuracy analysis [Sch1893-1]. In June 1893 Kloth [Klo1893] replied in the same journal, defending his solution. Ultimately, the success of the device over many decades can be seen as a vindication of Kloth's idea (Figure 35). For decades, these plates came with a one-page set of instructions on how to use the device to determine the area of quadrangles.

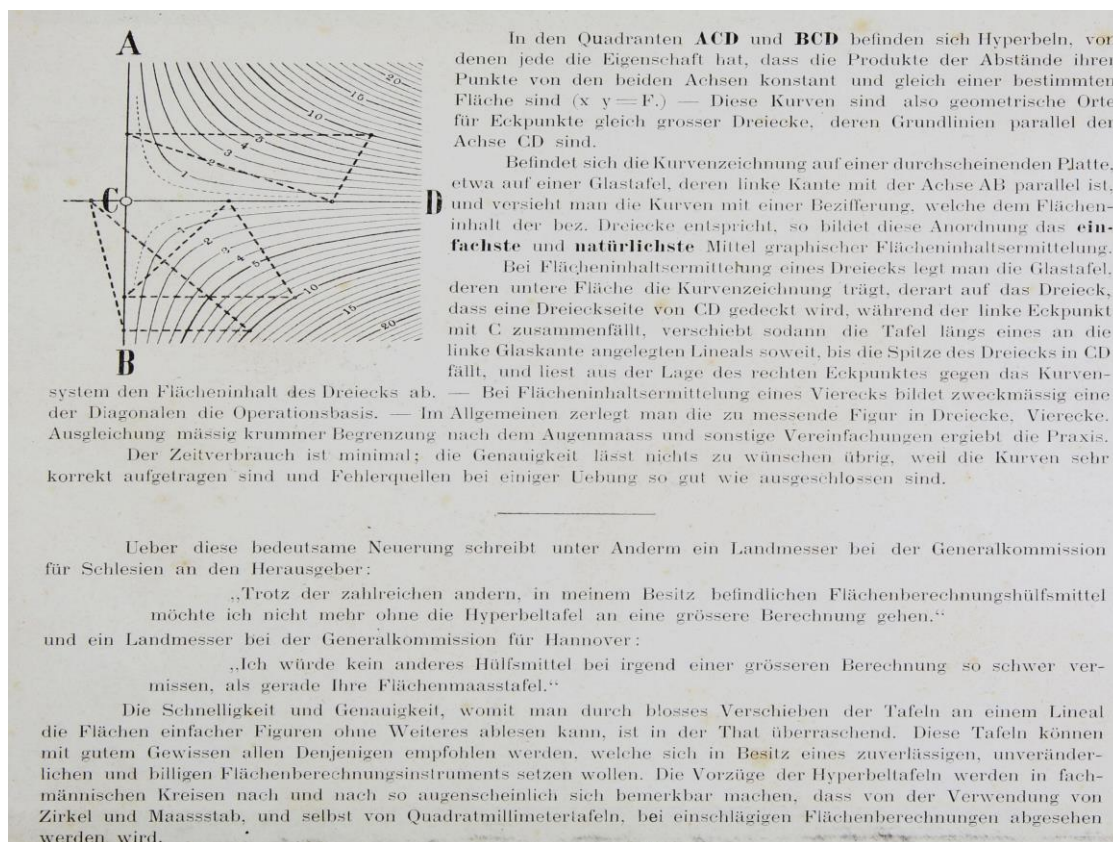


Figure 36: One page manual coming with above shown Kloth's Hyperbolic Table (Source: [Cor2023], courtesy www.mathinstruments.ch),

31. von Erichsen's Planimeter, USA, 1884

Another simple mechanical realisation of a harp planimeter had been introduced by Paul von Erichsen in 1883 or 1884 [Eri1884]. Again the well-known strip measuring approach got mechanized. The metal frame device is placed over the figure to be measured. On the metal frame, a scale has been mounted between serrated ledges. Pressing on a spring mounted knob **d** (Figure 37), the scale moved one tooth forward on the ledges and therefore moves in constant steps over the figure. For each scale position, in the corresponding strip, the distances between the boundaries of the figure are measured with the measurement wheel, guided by the scale. The measurement wheel is constructed in such a manner that the wheel itself moves along the centre of the strip. The distances summarized on the wheel times the strip width gives the total area of the figure. Depending of the scale of the figure the total result might need to be multiplied or divided by a corresponding scaling factor. In the article [Eri1884] the author mentioned that this planimeter had been in use at the offices of the US Coast and

Geodetic Survey in Washington, D.C. It was also advertised as follows: *“It is easy to see that, theoretically, this method guarantees greater accuracy in the calculation of the area than if the outlines of the figure are circumscribed, as is the case with Amsler’s planimeter”*. Unfortunately, the author of the article gave no other indication of its accuracy. Paul von Erichsen was born 21st November 1832 in Germany. He was found dead in his room aged 82 years, after leaving his workplace on 21st December 1914. He was unmarried and had no relatives in the USA [NN1914]

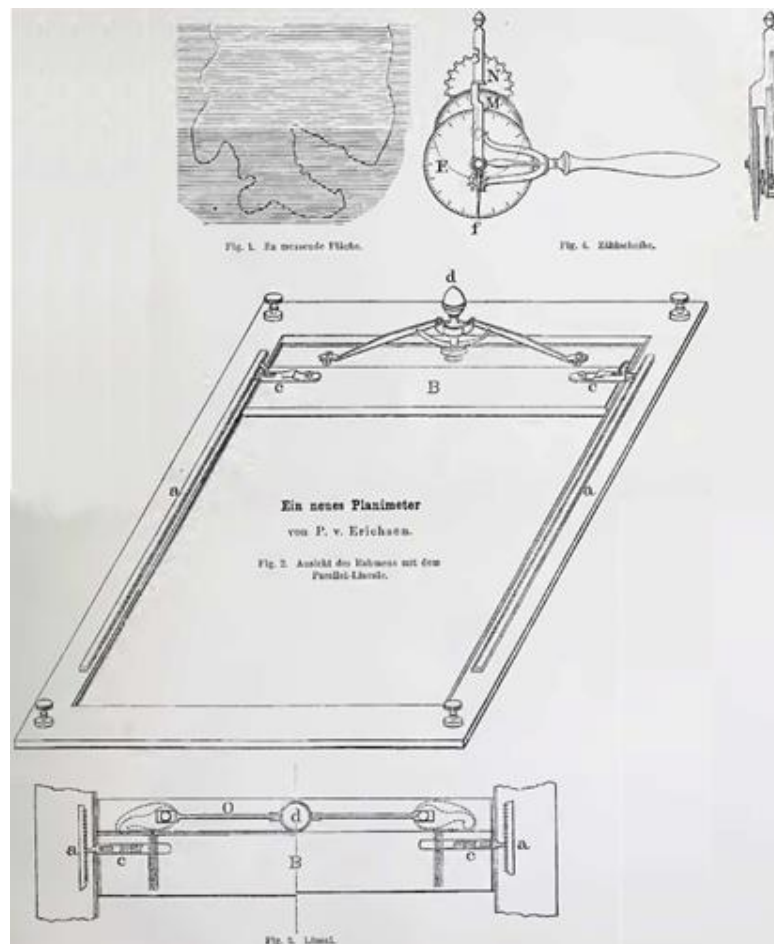


Figure 37: Erichsen's Planimeter from [Eri1884]

32. Updated timeline of early area measurement instruments

Above sections provide an overview of many area measurement devices. In an earlier article Durand-Richard [Dur2010] stated: *“All the productions mentioned above, even those more isolated and now missing – such as planimeters by Josef Stadler, at Eisenerz (Austria), in 1855; by Gierer, at Fürth; by Decher, at Augsburg in 1856; by Bleranovski and Bouniakovsky, in 1852 and 1856, and later A. Krylov in 1908 at Saint-Petersburgh demonstrated the significant circulation of technical knowledge linked to industrial development taking place within European countries in the 19th century.”* Whilst this statement is definitely true, it can be extended to devices by inventors such as those discussed in this paper. With this overview we have tried to expand on the information given in the previous articles on this subject [DreHae2009-1], [DreHae2011-1] and [Zer2021-4], as many additional references to this exciting topic can be found in [Zer2012] and the corresponding online database. Of course, the overview cannot be complete. This is sometimes due to missing or unavailable documentation. But also due to language barriers of the authors, inventions and thus inventors from other parts of the world (e.g. Russia, China, Japan, etc.) are not covered at all. Nevertheless, with the available information from this article, not to mention the bibliography, we were able to construct a substantially updated version of the timeline introduced in [Zer2021-4].

Year	Inventor; country; device name; principle	References
1810	1814 Herman; Germany	[Tru1865] p. 202
	1814 Bryon; France; Planimetre Graphique; Geometrical transformation	[Bry1814]
	1815 Oldenburg; Germany	[Pie1825]
	1815 Gelinski; France; Calculateur; Geometrical transformation	[Gel1815]
	1815 Zobel-Müller; Germany; Mechanized harp planimeter	[MulZob1815]
	1817 Zareba; Poland; Geometrical transformation	[Zar1829]
	1819 Laur; France; Olarithme; Hyperbola graph	[Lau1855]
	1820 Colberg; Poland; Geometrical transformation	[Col1824] , [Col1825] , [Col1820-1]
	1821 Wagner; Germany; Triangle planimeter	[Wag1821]
	1823 Posener; Austria	[Pos1823]
1820	1823 Wießner; Germany; Ttriangle planimeter	[Wie1823]
	1824 Gonella; Italy	[Gon1825]
	1824 Harkort; England	[Har1824-1] , [Rud2009-10]
	1825 Oldendorp; Germany; Harp planimeter	[Sch1832]
	1825 ? Alder; Austria; Harp planimeter	[Lem1849] , [Ams1907-1]
	1825 Netto; Germany	[Net1825]
	1826 Bryon; France; Improved Planimetre Graphique	[Bry1826]
	1826 Westfeld; Germany; Ringmesser; Ring measurements	[Wes1826]
	1826 Schmidt; Germany; Triangle planimeter	[Sch1827-1]
	1827 Oppikofer; Schweiz; Manufactured by Pfäffli	[Ams1856-6]
1828	1828 Wießner; Germany; Rechnungsuhr (calculating clock)	[Wie1828]
	1828 Tenner; Germany; Inhaltsmesser (content meter); Geometrical transformation	[Ten1828]
1830	1831 Littmark; Sweden; Pollettcirkel; Strip measuring	[Lit1831] , [Lit1857] , [Ber1863] page 15 , [Gus2007]
	1833 Ernst (1833/1836); France; Cone wheel	[Bau1853-1]
	1835 Stabrowsky; Prussia	[NN1835-1] , [Sta1835p]
	1839 Mason ?; England; Computing scale; Strip measuring	[Mas1841] , [Bou1845] , [Bru1840-2]
	1839 Lalanne; France; Arithmo-planimètre; Cone wheel	[Lal1840] & figures
1840	1841 Miller; England; Pantoscale ; Multi purpose instrument	[Mil1842]
	1842 Horsky; Austria; Triangle planimeter	[Wag1842] , [Hun1862]
	1844 Beuvière; France	[Mor1846]
	1844 Dupuit; France; Roulette de Dupuit; Strip measuring	[Dup1844]
	1846 Hennon; France; Polygonometre; Geometrical transformation	[Hen1846]
	1848 Godfray; England; Argometer	[God1848]
	1849 Wetli; Switzerland	[Sta1850-1] , [Bau1853-1]
	1849 Hansen; Germany; Based on Wetli planimeter	[Bau1853-1]
	1849 Starke & Kammerer; Austria	
1850	1850 Pfeier	
	1850 Sang; Scotland; Planometer, later platometer	[San1851] , [San1852]
	1852 Burman; Based on Wetli planimeter	[Bur1852p]
	1852 Baranowsky; Poland	[Kuc1902]
	< 1855 Fichtbauer; Germany	[Bau1855]
	< 1855 Keller; Italy	[Bau1855]

1855	Bouniakowski; Poland; Planimetre-pantographe	[Sau1856] , [Bou1855] & figures
	Decher; Germany; Based on Wetli planimeter	[Dec1855]
	Hamann; Harp planimeter	
	Hermann (rediscovered); Germany	[Tru1865] p. 202
	Stadler; Austria; Hyperbel planimeter; Strip measuring	[Sta1858] , [Dol190?]
	Dasnoy; Belgium; Calculateur-Rapporteur; Geometrical transformation	[Das1856]
1856	Gangloff; Bohemia; Geometrical transformation	[Gan1857] , [LeiSimSimWoi2020]
1860	1861 Liedbeck; Sweden; Mechanized harp planimeter	[Lie1861p]
	< 1862 Hogrewe; Glass planimeter	[Hun1862]
	> 1861 <	
	1880 Köhler; Sweden; Mechanized harp planimeter	
	1865 Trunk; Germany	[Tru1865]
	1867 ?; Netherlands; Transformateur; Geometrical transformation	[Bar1867]
1870	1870 Ljungström; Sweden; Cirkelplanimeter	[Lju1870p]
	1874 Coradi & Ott; Switzerland	https://www.mathinstruments.ch
	Hofmann; Germany; Hofmann's instrument; Geometrical transformation	[Dol1874] , [Hof1874]
	~1875 Prytz; Danish; Hatchet planimeter	[Pry1886]
	1876 Hohmann; Germany	[Cor1881]
	Schlesinger; Austria; Tachygraph Planimeter; Geometrical transformation	[Sch1877]
1880	Jednoduchý; Poland; Jednoduchý's planimeter; Geometrical transformation	[Gan1878] , [Lei2020]
	1883 Kloth; Germany; Hyperbolic plates; Triangle planimeter	[Klo1883p] , [Wil1892-2] , [Klo1893]
	1884 Erichsen; USA; Strip measuring	[Eri1884]
	1889 Fennel; Germany; Area measurement for triangles; Triangle planimeter	[Fen1889p]
	1894 Mönkemöller; Germany; Strip measuring	[Mon1894p]
	~1899 Berg; Sweden; Mechanized harp planimeter	
1900	< 1902 Majewski; Poland	[Kuc1902]
	1910 de Wal; Harp planimeter	[Wal1910] , [Wal1910p]
1910	1910 Schnöckel; Germany; Optical planimeter	[Sch1910-04p]

Please see the separate quite incredible nine-page Bibliography available on Gazette Plus in www.UKSRC.org.uk

This article, together with its precursor: “Early Area Measurement Instruments” from Gazette 21; plus both Bibliographies, will be combined into one “Area Measurement Instruments which will also be published later and available for download on www.UKSRC.org.uk

Bibliography

Note: All Internet links tested and working as of 9th July 2023.

[Ams1907-1] Amsler, A.: **Das Planimeter und seine Erfindung**, Zeitschrift des Vereins Schweizer Konkordatsgeometer, Vol. 5, no. 7, 1907, pages 117-122. Note: See also part 2 in [Ams1907-2]. Link: <https://www.e-periodica.ch/digbib/view?pid=geo-001:1907:5::328#123>

[Ams1907-2] Amsler, A.: **Das Planimeter und seine Erfindung**, Zeitschrift des Vereins Schweizer Konkordatsgeometer, Vol. 5, no. 8, 1907, pages 125-132. See also part 1 in [Ams1907-1]. Link: <https://www.e-periodica.ch/digbib/view?pid=geo-001:1907:5::342#131>

[And1876] Andersson, J. G.: **Geodetisk Mättningskunskap**, Albert Bonniers Förlag, Stockholm, 1876, on pages 223-244 an overview of planimeters is given, especially about Liedbeck's harp planimeter and Ljungström's planimeters. Link: <http://runeberg.org/geodet/0231.html>

[Bar1867] Barenbroek, E.: **Beginnelsen der Werkdadige Meetkunst**, Groningen, 1867. Note: On page 116 a description of Laport's Transformateur is given.

[Ber1863] Bergstrand, Emanuel: **Anteckningar i fältmättningskonst, grafisk geodesi, utgifna för nybegynnare**. Edition 2, 1863. Note: Page 15 describes Littmarck's Pollettcirkel. Link: https://www.google.de/books/edition/Anteckningar_i_f%C3%A4ltningskonst_grafi/M7Yol4exeFYC

[Bou1843] Bourns, Charles: **The Principles and Practice of Land, Engineering and Other Surveying**, John Olliver, London, 1843, pages 167. Link: https://www.google.de/books/edition/The_Principles_and_Practice_of_Land_Engi/Jx5kAAAAcAAJ

[Bou1845] Bourns, Charles: **Principles and Practice of Surveying: Land and Engineering, Trigonometrical, Subterraneous and Marine**. Edition 2, 1845. Note: Pages 166-169 describe the Computing Scale, modified by Elliott. The inventor of the original item is not known. Link: <https://ia600808.us.archive.org/23/items/principlespracti00bour/principlespracti00bour.pdf>

[Bou1855] Bouniakovsky: **Note sur un nouveau planimètre: par M. Bouniakovsky**, Eggers et Comp., St.-Petersbourg et Léopold Voss, Leipzig, Bulletin de la Classe Physico-Mathématique de L'Académie Impériale des Sciences de St.-Petersbourg, Vol. XII, 10th August, 1858, pages 132-137. Link: <https://www.biodiversitylibrary.org/item/173056#page/574/mode/1up>, figures: <https://www.biodiversitylibrary.org/item/173056#page/729/mode/1up>

[Bru1840-1] Bruff, Peter: **A Treatise on Engineering Field-Work comprising the Practice of Surveying, Levelling, Laying Out Works, and other Field Operations connected with Engineering**, Simpkin, Marshall, and Co., Stationers' Hall Court, London. Edition 2, 1840, pages 140-142. Link: <https://books.google.de/books?id=ntLOM6WbD2YC>

[Bru1840-2] Bruff, Peter: **Remarks on the New Scaling Instrument**, The Civil Engineer and Architect's Journal, 1840, pages 405. Link: https://www.google.de/books/edition/The_Civil_Engineer_and_Architect_s_Journ/GLxBAAAAYAAJ

[Bry1814] Bryon, Charles-Henri: **Nouvelle Méthode de Calculer les Surfaces, et Description du Planimètre Graphique**, C. A. Delhorme, Imprimeur de la Préfecture du Jura July, 1814. Note: 24 pages.

[Bry1826] Bryon, Charles-Henri: **Instruction Sommaire sur l'Usage du Planimètre Graphique, Instrument imaginé pour donner au Calcul des Surfaces plus de célérité et de précision**, Gerard, Épinal, 1826. Note: 14 pages.

[Bur1852p] Burman, Lars Abraham: **å planimetrar, enligt ritning**, Swedish patent number X983. Application filed 26.11.1852. Patent accepted 26.11.1852. Note: See also the improvement in [BurCav1856p].

[BurCav1856p] Burman, Lars Abraham and Cavallin, C.: **å förbättring af den för Burman den 26 November 1852 patenterade planimetér**, Swedish patent number X1258. Application filed 23.09.1856. Patent accepted 23.09.1856. Note: Improvement of [Bur1852p].

[Cen1883] Central-Comité: **Catalog der Ausstellung culturhistorischer Gegenstände veranstaltet zur Feier der 600jährigen Regierung des Hauses Habsburg in Steiermark**, Verlag des Central-Comités, Graz July, 1883, Industriehalle. Note: Page 126 mentions two presented variants of Stadler's hyperbolic planimeter, as well as two planimeters by Albert Miller von Hauenfels.

Link: https://www.google.de/books/edition/Catalog_der_Ausstellung_culturhistorisch/6z6Pc3OL4i8C

[Col1820-1] Colberg, Juliusz: **Opisanie składu i użycia Plenimetru, nowo wynalezionego mierniczego narzędzia do dochodzenia powierzchni płaskich**, W. Drukarni N. Glücksberga, no. 2, 1820, pages 34-54. Note: First publication of Colberg's planimeter. Translation of the title: Describing the composition and use of the planimeter, a newly invented measuring tool for investigating flat surfaces.

Link: <https://www.digitale-sammlungen.de/en/view/bsb10300728?page=232,233>

[Col1824] Colberg, Juliusz: **Sposób dochodzenia powierzchni płaskich bez użycia rachunku za pomocą nowo - wynalezionego instrumentu planimetr zwanego, albo za pomocą w tym celu urządzonych Tabell. Dla użytku jeometrycznych**, Drukarni Jego Ces. Król. Mosci Rządowéy, Warszawa, 1824. Note: 64 pages. See also the German translation in [Col1825].

Link: <https://crispa.uw.edu.pl/object/files/413862/display/Default>

[Col1825] Colberg, Juliusz: **Anweisung den Inhalt ebener Flächen ohne Rechnung genau zu finden, und die Theilung der Figuren zu erleichtern, vermittelt eines neu erfundenen Instruments des Planimeters... aus dem polnischen Forst-Journale "Sylvan" übersetzt. Mit einer Vorrede von Dr. J. P. Gröson. Nebst 4 Kupfertafeln**, Ludwig Dehmigke, Berlin, 1825. Note: 46 pages. German translation of [Col1824].

Link: <https://www.digitale-sammlungen.de/en/view/bsb10081608>

[Cor2023] Coradi, Markus: **Hyperbeltafel**. The internet pages show two Kloth's hyperbolic plates, one from 1949. Link: <https://www.mathinstruments.ch/de/gallery/inv-9.024.html>

[Das1856] Dasnoy, Jean-Baptiste: **Instruction sur l'usage du Calculateur=Rapporteur de Gelinsky, perfectionné par Dasnoy, Géomètre du Cadastre, inventuer du Cathétographe, de l'Additionneuse á clavier, etc.**, Typographie et Lithographie de J. Bourger, Arlon, 1856. Note: 48 pages manual.

Link: <https://opac.kbr.be/LIBRARY/doc/SYRACUSE/13404781>

[Dec1855] Decher, G.: **Theorie und Construction eines neuen, auf Polar-Coordinaten gegründeten Planimeters**. Dinglers Polytechnisches Journal, Vol. 136, 1855, pages 168-181.

Link: <http://www.polytechnischesjournal.de/articles/ar136041.html>

[Dec1856-1] Decher, G.: **Ueber zwei neue, auf die Berechnung von Polarcoordinaten sich stützende Planimeter von Bouniakovsky in Petersburg und J. Amsler in Schaffhausen, nebst einer wesentlichen Modification des erstern von Professor G. Decher**, Dingler's Polytechnisches Journal, Vol. 140, 1856, pages 27-42, Tafel I, Fig. 5-10. Link: <http://www.polytechnischesjournal.de/articles/ar140007.html>

[Dol1874] Doll, M.: **Instrument zur Verwandlung von Vielecken in Dreiecke durch Parallelabschieben**, Zeitschrift für Vermessungswesen, Vol. 3, no. 2, 1874, pages 83-85. Note: See also the addendum in [Hof1874].

Link: https://archive.org/details/bub_gb_7Y1IAAAAMAAJ

[Dol2018] Dolata, T.: **Patent legislation of the Kingdom of Poland from 1817 to 1867**, Journal on the European History of Law, Vol. 9, January, 2018, pages 104-109. Abstract Link: https://www.researchgate.net/publication/329759537_Patent_legislation_of_the_Kingdom_of_Poland_from_1817_to_1867

[Dre2010] Drechsler, Stefan: **The Planimeter of J. Gierer. Invented by Misunderstanding**, IM2010 Proceedings, 16th International Meeting of Collectors of Historical Calculating Instruments, Dutch Circle for Historical Calculating Instruments, 2010, pages 165-173. Link: <https://rechnerlexikon.de/files/drechsler2010.pdf>

[DreHae2009-1] Drechsler, Stefan and Haeberlin, Barbara: **Prinzipien und Konstrukteure nichtintegrierender Planimeter, ein historischer Überblick**, 15. Internationales Treffen der Rechenschiebersammler und 4. Symposium zur Entwicklung der Rechentechnik, Ernst Moritz Arndt Universität Greifswald, 2009, pages 97-116. Link:

[https://www.researchgate.net/publication/263333775 Prinzipien und Konstrukteure nichtintegrierender Planimeter ein historischer Überblick](https://www.researchgate.net/publication/263333775_Prinzipien_und_Konstrukteure_nichtintegrierender_Planimeter_ein_historischer_Ueberblick)

[DreHae2011-1] Drechsler, Stefan and Haeberlin, Barbara: **Cones, Disks, Wheels and Spheres for Area and Integration from Bavaria to Boston and Beyond**, Proceedings IM 2011. From the Old World to the New: European Roots of American Slide Rules. Technology Advances from the Old World to the New. 17th International Meeting for Collectos of Historical Calculating Instruments, 2011, pages 174-194. Link: <https://www.rechnerlexikon.de/files/drehae2011.pdf>

[Dup1844] Dupuit, Jules: **Sur un instrument destiné á calculer rapidement toutes les surfaces et en particulier celles de déblai et remblai**, Carilian-Goeury et Ver Dalmont, Paris, Annales des Ponts et Chaussées, no. Tombe VIII, 2e serie, 2e semestre, 1844, pages 122-132. Link: <https://books.google.co.uk/books?id=sTV9Bl72rSMC&pg=PA122>

[Dyc1892] Dyck, W. v.: **Katalog mathematischer und mathematisch-physikalischer Modelle, Apparate und Instrumente**, Königliche Hof- und Universitäts-Buchdruckerei Dr. C. Wolf & Sohn, München, 1892. Note: 430 pages, many figures, (Reprint 1994, George Olms Verlag, incl. Supplement). Link: https://cnum.cnam.fr/PDF/cnum_M9874.pdf, https://www.google.de/books/edition/Katalog_mathematischer_und_mathematisch/GFnkAAAAAAAJ

[Eks1903] Ekstrand, Viktor: **Svenska landtmätare 1628-1900 biografisk förteckning**, Umea & Uppsala, 1903. Link: <https://archive.org/details/SvenskaLandtmatare/page/n7/mode/2up>

[Ell1925] Elling: **The method of area from coordinates was reformulated in an equation by the Danish land surveyor Elling in 1925**, Tidsskrift for Opmaalings- og Matrikulsvaesen, 1925, page 305.

[Eri1884] von Erichsen, Paul: **Ein neues Planimeter**, Ministerium der Öffentlichen Arbeiten, Verlag von Ernst & Korn, Berlin, Centralblatt der Bauverwaltung, Vol. 4, no. 42, 18th October, 1884, pages 431. Link: https://digital.zlb.de/viewer/image/14688302_1884/441/

[Fav1873] Favaro, Anton: **Beiträge zur Geschichte der Planimeter**, Wien, Allgemeine Bauzeitung, Vol. 38, 1873, pages 68-90, 93-108. Link: <https://books.google.de/books?id=O7BctqDFJUUC&pg=PA1>, <https://www.e-rara.ch/zut/content/zoom/28249349>

[FouPla1838] Place and Foucard: **Livre de l'arpenteur-géomètre, Guide Pratique de l'Arpentage et du Lever des Plans**, Paris, 1838, pages 97, Fig. 54 and 55. Link: <https://gallica.bnf.fr/ark:/12148/bpt6k1080448.image>

[Gan1857] Gangloff: **Kurze Beschreibung und Gebrauchs=Anleitung eines einfachen Flächenberechnungs=Apparates**, Verhandlungen der mährisch-schlesischen Forstsection, no. 26, 1857, pages 57-62. Note: See also [NN1857-4]. Link: https://books.google.de/books/download/Verhandlungen_der_Forst_Section_f%C3%BCr_M%C3%A4.pdf?id=xtdthb7Bzp8C

[Gel1815] Gelinski, Frederic Charles: **Description du Calculateur de M. Gelinsky, et manière de s'en servir. Par M. S****, Ingénieur-Vérificateur du Cadastre**, Impr. de Vve Bizette, 1815. Note: Gelinski's pamphlet is listed in several bibliographies but no copy found until now.

[Gie1854] Gierer, Johann: Entwurf eines Planimeters, mit welchem man den Quadratinhalt ebener Figuren nach Kreisausschnittelementen oder auch nach Ringelementen ausmessen kann, Fürth, Jul. Volkhard, 1854.

[God1848] Godfray, Hugh: Godfray's Argometers, Mechanics' Magazine, 19th February, 1848, pages 169-174. Link: <https://anno.onb.ac.at/cgi-content/anno?aid=mem&datum=18480219&zoom=33>

[Gra1908-1] Grabowski, L.: Über einige Planimeter polnischer Erfindung, insbesondere über das Planimeter von Zaręba, Organ des Vereines der Österreichischen K. K. Vermessungsbeamten, Österreichische Zeitschrift für Vermessungswesen, Vol. 6, no. 3, 1st March, 1908, pages 70-74. Note: See also part 2 in [Gra1908-2]. Link: <https://anno.onb.ac.at/cgi-content/anno?aid=ovm&datum=19080301&zoom=33>

[Gra1908-2] Grabowski, L.: Über einige Planimeter polnischer Erfindung, insbesondere über das Planimeter von Zaręba, Organ des Vereines der Österreichischen K. K. Vermessungsbeamten, Österreichische Zeitschrift für Vermessungswesen, Vol. 6, no. 4, 1st April, 1908, pages 108-112. Note: Part 2 of [Gra1908-1]. Link: <https://anno.onb.ac.at/cgi-content/anno?aid=ovm&datum=19080401&zoom=33>

[Gre2023-1] Green, David: Personal communication, April 2023.

[Gre2023-2] Green, David: Planimeter Vault: 5. Computing Scales.

Link: <https://www.uraone.com/PlanimeterVault/hair/>

[Hae2011] Haeberlin, Barbara: Stanley computing scale. Rechnerlexikon, 2011. Link: https://www.rechnerlexikon.de/artikel/Stanley_Computing_Scale

[Hau1858] von Hauer, Julius: Ueber die neuen, auf die Berechnung mit Polar=Coordinationen sich stützende Planimeter, Verlag von Friedrich Manz, Wien, Erfahrungen im berg= und hüttenmännischen Maschinen=, Bau= und Aufbereitungswesen, Zusammengestellt aus den amtlichen Berichten der k. k. österr. Berg=, Hütten= und Salinen=Beamten, 1858, pages 39-42, Tafeln VII. Link: <https://www.digitale-sammlungen.de/en/view/bsb10705800?page=45>, figures: <https://www.digitale-sammlungen.de/en/view/bsb10705819?page=28>

[Hen1846] Hennon, V.: Géodésie des Forêts, Nevers. Edition 1, 1846. Note: Further edition: 2 (1851), Mme Ve Bouchard-Huzard, Paris. Complete title of 2nd edition: Géodésie Pratique des Forêts, a l'Usage des Agents Forestiers, Propriétaires, Régisseurs, Agents Voyers, et autres Personnes d'Occupant de l'Estimation et de l'Aménagement des Bois. Link: 1st edition: <https://gallica.bnf.fr/ark:/12148/bpt6k9708068k>. 2nd edition: <https://gallica.bnf.fr/ark:/12148/bpt6k96006841>

[Hin1883] Hinterhölzl, A.: Auch ein Beitrag zur Geschichte der Planimeter, Zeitschrift für Vermessungswesen, Vol. 12, 1883, pages 164-167. Link: <https://archive.org/details/zeitschriftfrve09unkngoog>

[Hof1874] Hofmann: Nachtrag zu dem Instrumente zur Verwandlung von Vielecken in Dreiecke, Zeitschrift für Vermessungswesen, Vol. 3, 1874, pages 124. Note: Addendum of [Dol1874]. Link: https://archive.org/details/bub_gb_7Y1IAAAAMAAJ/page/n141/mode/2up

[JacPet1848] Jacobi, M. H. and Peters, C. A. F.: РУЧНОМЪ ПЛАНИМЕТРЪ ЕРМАКОВА (Jermakow's Handheld Planimeter), Akademii'a' nauk SSSR, Prizuzhdenie uchrezhdennykh P.N. Demidovym nagrad, Vol. 17-19, 20th April, 1848, pages 273-280. Link: https://www.google.de/books/edition/Otchet_ImpertorskogoC4%AD_S_PeterburgskoC4%AD_a/qz1FAAAIAAJ?hl=en&gbpv=1&pg=RA1-PA273

[Jer2023] Jerripedia: Hugh Godfray. Link: https://www.theislandwiki.org/index.php/Hugh_Godfray

[Jor1890] Jordan, W.: Handbuch der Vermessungskunde, J. B. Metzlerscher Verlag, Stuttgart. Three volumes. Further edition: 5 (1897); 8 (1914), revised edition by O. Eggert. Volume 2 of 5th Edition Link: https://www.google.de/books/edition/Feld_und_land_messung_5_verb_und_erweite/mqgRAAAAYAAJ?hl=en&gbpv=1&dq=Vermessungsinspektor+Hofmann&pg=PA86&printsec=frontcover

[Kal1849] Källström, Wilhelm Emanuel: **Produkt-och reciprok-tabeller, till multiplikations-och divisions-exempels, till reguladetri-frågors kortare och säkrare upplösning än på vanligt sätt samt till kontrollerande af på vanligt sätt erhållne uträknings-resultater, utg af**, Stockholm, 1849.

[Kal1852p] Källström, Wilhelm Emanuel: å s. k. **planimetrar**, Swedish patent number X930. Application filed 15.03.1852. Patent accepted 15.03.1852.

[Kal1852] Källström, Wilhelm Emanuel: **Om Patent Planimetern**, Stockholm, 1852.

[Klo1883p] Kloth, Max: **Transparente Maßstäbe und Rechentafeln, sowie das Verfahren zu ihrer Herstellung**, 8th August, 1883, Schleswig, [German patent number 26695](#). Published 18.04.1884

[Klo1893] Kloth, Max: **Zu den Artikeln über Kloth's Flächenmaasstafeln**, Zeitschrift für Vermessungswesen, Vol. 22, no. 11, 1st June, 1893, pages 338-340. Note: Response on [Wil1892-2, Sch1893-1]. Link: <https://archive.org/details/zeitschriftfrve23vermgoo/page/338/mode/2up>

[Kuc1902] Kucharzewski, Feliks: **Planimetry Polskie i ich wynalazcy**, Sklad W Ksiegarnie E. Wende I S-ka, Warszawa, 1902. Note: 46 pages on planimeter at Poland. Link: <https://polona.pl/item-view/4853f669-1425-4308-acf0-c18a5bb8dce9?page=4>, <https://delibra.bg.polsl.pl/Content/1236/planimetry.pdf>

[Lal1839-1] Lalanne, Léon: **Balance arithmétique ou nouvell machine á calcul**, Bachelier, Imprimeur-Libraire, Paris, Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences, Vol. IX, 2nd September, 1839, pages 319-322. Link: <https://www.biodiversitylibrary.org/item/20576#page/329/mode/1up>, a

[Lal1839-2] Lalanne, Léon: **Rapport sur une balance á calcul présentée**, Bachelier, Imprimeur-Libraire, Paris, Comptes Rendus Hebdomadaires des Séances de l'Académie des Sciences, Vol. IX, 2nd September, 1839, pages 693-694. Link: <https://www.biodiversitylibrary.org/item/20576#page/703/mode/1up>

[Lal1840] Lalanne, Léon: **Sur l'arithmoplanimètre, machine arithmétique et géométrique, donnant facilement les résultats des opérations les plus compliquées de calcul et de planimétrie**, Carilian-Goeury et V Dalmont, Paris, Annales des Ponts et Chaussées, 1re Série, 2e Semestre, 1840, pages 3-61. Link: <https://gallica.bnf.fr/ark:/12148/bpt6k408454g/f6.item>, figures: <https://gallica.bnf.fr/ark:/12148/bpt6k408454g/f406.item.r=arithmoplanimetre.zoom>

[Lal1876] Lalanne, Léon: **1839. Arithmoplanimètre**, Gauthier-Villars, Paris, Notice sur les travaux et titres scientifiques, 1876, pages 16-17. Link: https://www.google.de/books/edition/Candidatures_acad%C3%A9miques/DZO1aFuVTcoC?hl=en&gbpv=1&pg=RA14-PA16

[Lau1855] Laur, Jean-Antoine: **Géodésie pratique simplifiée et perfectionnée: á l'usage du génie civil et militaire, des ponts et chaussées et des mines**, Edition 6, 1855. Note: The appendix contains the description "Olarithme Français ou Planimètre-Laur Calculateur et Constructeur Graphique Universal". Link: <https://gallica.bnf.fr/ark:/12148/bpt6k3065969w>

[Lau1857-1] Laur, J. A.: **Vereinfachte und Vervollkommnete praktische Geodäsie**, Leipzig, Vol. 1, 1857. Link: <https://books.google.de/books?id=9WVCAAAAYAAJ>

[Lau1857-2] Laur, J. A.: **Vereinfachte und Vervollkommnete praktische Geodäsie**, Leipzig, Vol. 2, 1857. Link: https://www.google.de/books/edition/Vereinfachte_und_vervollkommnete_praktis/3t0HAAAacAAJ

[Lau1859] Laur, J. A.: **Notice sur les planimètres, ou Observations théoriques et pratiques sur l'origine** Jean-Antoine Laur, Vol. 1, 1859. Link: <https://gallica.bnf.fr/ark:/12148/bpt6k840633m.image>

[Lec1846] Lecoy, François: **Méthode simple et facile pour lever les plans**, Paris. Edition 16, 1846. Link: <https://www.e-rara.ch/zut/content/pageview/322796>

[Lie1861p] Liedbeck, N. F.: **Å tillverkning af ett slags instrument att begagnas vid beräkning af ytor**, Swedish patent number X1556. Application filed 14.01.1861. Note: Translation of title: Manufacturing of a kind of instrument to be used in calculation of surfaces.

[Lit1831] Littmark, Frederick: **Beskrifning öfwer Pollett-Cirkeln Ett Instrument för Kartors Uträknande**, Georg Scheutz, Stockholm, 1831. Note: Translated title: Description of the Pollett Circle: An instrument for calculating maps.

[Lit1857] Littmark, Frederick: **Nya Pollettcirkelns begagnande till Kartors Uträkning enligt Nya och på samma gång Gamla Ytmåttet. Ny lättfattlig beskrifning**, Westerås, Sweden, 1857. Note: Translated title: The use of the new policy circle for the calculation of maps according to the new and at the same time old surface measurement. New easy-to-understand description.

[Lju1870p] Ljungström, J. P.: **Cirkelplanimeter (?)**, Swedish patent number ?. Application filed 1870. Note: Cited in [NN1870].

[Mal1702] Mallet, Allain Manesson: **La Geometrie Pratique (Contenant La Planimétrie, ou la mesure des Superficies (de qu'on appelle l'Arpentage) avec les Methodes de transfigurer, d'augmenter, & de diviser toutes fortes de Terre, Bois, &c)**, Paris, Vol. 3, 1702. Link: https://www.google.de/books/edition/LA_GEOMETRIE_PRATIQUE/njTrE8_EhfkC.

[Mas1841] Mason, Robert: **The New Scaling Instruments**, The Surveyor, Engineer and Architect, 1841, pages 171.

Link: https://www.google.de/books/edition/The_Surveyor_Engineer_and_Architect/TqoEAAAAQAAJ

[Mer1863] Merrett, H. S.: **A Practical Treatise on the Science of Land and Engineering Surveying, Levelling, Estimating Quantities, &c. with a General Description of the several Instruments required for Surveying, Levelling, Plotting, &c.**, Chapter: "H. S. Merrett's Patent Improved Computation Scale", E. & F. N. Spon, London, 1863, pages 261, plate 41.

Link: https://www.google.de/books/edition/A_practical_treatise_on_the_science_of_L/rpNWAAAACAAJ

[Mil1842] Miller, John: **Description d'un instrument nommé pantoscale, destiné à rapporter, à copier et à réduire des plans, à calculer les surfaces, etc.**, Publié avec l'approbation de M. le Ministre de l'Agriculture et du Commerce, Bulletin de la Société d'Encouragement pour l'Industrie Nationale, Vol. 41, 1842, pages 7-9.

Link: <https://cnum.cnam.fr/CGI/fpage.cgi?BSPI.41/10/100/628/6/525>,

figures: <https://cnum.cnam.fr/CGI/fpage.cgi?BSPI.41/11/100/628/6/52>

[Mor1910] Morin, H.: **Catalogue Général**, Paris. Edition 37, 1910-1911. Page 108 shows a Roulette Dupuit.

Link: <https://cnum.cnam.fr/pgi/redirect.php?ident=M11123>,

[Mul1879] Müller, Franz: **Die Planimeter von Gangloff und Schlesinger**, Zeitschrift für Vermessungswesen, Vol. 8, 1879, pages 150-169. Link: <https://www.digitale-sammlungen.de/de/view/bsb11390637?page=162>

[NN1815] n. n.: **L'Imprimerie Impériale, Bulletin des lois de la République Française**, 5e Série, Vol. Tome Troisième, no. No 691 April, 1815, pages 104. Note: Charles-Frederic Gelinski appointed Ingénieur-Vérificateur du Cadastre by Royal decree. Link: <https://books.google.de/books?id=K11fAAAAcAAJ>

[NN1829-2] n. n.: **Neueste Literatur**, Didastalia oder Blätter für Geist, Gemüth und Publizität, Vol. 7, no. 254, 11th September, 1829. Note: Refers to Tenner's planimeter and cites [Ten1828]. Link: <https://www.digitale-sammlungen.de/en/view/bsb10531033?page=296>

[NN1829-3] n. n.: **Polen. Warschau, 15. Juni**, Nürnberg, Der Friedens und Kriegs Kurier, Vol. 155, no. 174, 22rd June, 1829, pages 2. Note: Remark on extended Zareba patent from 1817.

Link:

https://www.google.de/books/edition/Der_Friedens_u_Kriegs_Kurier/pNZMAAAACAAJ?hl=en&gbpv=1&dq=zareba+patent%C2%A0&pg=PA7&printsec=frontcover

[NN1835-1] n. n.: **Patent-Ertheilungen**, Düsseldorf, Amtsblatt der Regierung zu Düsseldorf, no. 71, 14th November, 1835, pages 518. Note: Cites Stabrowsky's planimeter patent [Sta1835p]. Link: https://www.digitale-sammlungen.de/en/view/bsb10001223_00615_u001?page=2&q=%28%22patent+planimeter%22%7E10%29,

[NN1840-1] n. n.: **Lalanne's Arithmo=Planimeter**, Dinger's Polytechnisches Journal, 1840, pages 239. Link: <http://dingler.culture.hu-berlin.de/journal/page/pi075?p=00000257>,
<https://anno.onb.ac.at/cgi-content/anno-plus?aid=ptj&datum=18400004&seite=239>

[NN1845-2] n. n.: **Ueber das Planimeter von Dupuit**, Weidmann'sche Buchhandlung, Leipzig, Polytechnisches Centralblatt, Vol. 6, 1845, pages 76-78. Link: <https://books.google.de/books?id=IbsbAQAAIAAJ&pg=PA76>

[NN1850-1] n. n.: **De Calculateur Gelinski of Graphite**, Gebroeders Diederichs, Amsterdam, Nieuw Nederlandsch Magazijn, 1850, pages 404-405. Link: https://www.delpher.nl/nl/boeken1/gview?&identificer=B_5TAAAAcAAJ

[NN1856-2] n. n.: **Stockholm, den 25 Sept.**, Aftonbladet, no. 224, 25th September, 1856, pages 1-2. Note: Cites Burman's planimeter patent [Bur1856p]. Link: <https://tidningar-beta.kb.se/4345612/1856-09-25/edition/0/part/1/page/2?q=planimeter>

[NN1856-4] n. n.: **Welche neuester Zeit erfundenen forstlichen Maschinen, Geräte, Werkzeuge und Instrumente bewähren sich in der Praxis - und welcher Ergebnisse zeigt ihre Anwendung?**, Verhandlungen der mährisch-schlesischen Forstsection, no. 25, 1856, pages 106. Note on the demonstration of Gangloff planimeter. See also [Gan1857]. Link: <https://books.google.de/books?id=NPuauSFSH0C>

[NN1867] n. n.: **Mathematical drawing instruments**, Robertson, Broome, & Co., "Mechanics' Magazine" and Patent Offices, London, Mechanics' Magazine and Journal of Engineering, Agricultural Machinery, Manufactures, and Shipbuilding, New Series, Vol. 17, 8th March, 1867, pages 145-147. Note: About Stanley's instruments. Link: https://www.google.de/books/edition/Mechanics_Magazine/TCdJAAAAcAAJ

[NN1870] n. n.: **Mekanisk uppfinning**, Aftonbladet, 22nd April, 1879, pages 2. Note: Cites J. P. Ljungström's cirkel planimeter [Lju1870p]. Link: <https://tidningar-beta.kb.se/4345612/1870-04-23/edition/0/part/1/page/2?q=cirkelplanimeter>

[NN1872-2] n. n.: **Josef Stadler, Nekrolog**, Verlag der G. J. Manz'schen Buchhandlung, Wien, Österreichische Zeitschrift für Berg- und Hüttenwesen, Vol. 20, no. 8, 19th February, 1872, pages 60-61. Link: https://www.google.de/books/edition/Oesterreichische_Zeitschrift_für_Berg_u/VYAAAAAAMAAJ

[NN1875-1] n. n.: **209. Roulette Dupuit**, Arpentage Levé des Plans et Nivellement, Cours Élémentaire de Mathématique, 1875, pages 137-138. Link: <https://gallica.bnf.fr/ark:/12148/bpt6k937463x/f160.item>,

[NN1883-1] n. n.: **Polytechnischer Club in Graz**, Druckerei des Sports (FR. v. Karst), Wien, III., Seidl'sche 15, Der Bautechniker, Centralorgan für das österreichische Bauwesen, Vol. 3, no. 7, 16th February, 1883, pages 63. Link: <https://anno.onb.ac.at/cgi-content/anno-plus?aid=bau&datum=1883&page=71>

[NN1884-6] n. n.: **Personalnachricht**, Zeitschrift für Vermessungswesen, Vol. 13, no. 18, 1st September, 1884, pages 436. Note: Obituary Karl Hofmann. Link: https://archive.org/details/kwittwer_zeitschriftfurvermessungswesen_13_jahr_1884/page/436/mode/2up

[NN1887-2] n. n.: **Computing scale**, The Builder, Vol. 53, no. 2334, 29th October, 1887, pages 618-619. Note: About Merrett's Improved Computing Scale. Link: <https://babel.hathitrust.org/cgi/pt?id=njp.32101079220883&view=1up&seq=658>

[NN1892-6] n. n.: **Léon Lalanne**, L'Année Scientifique et Industrielle, Vol. 36, 1892, pages 588-589. Link: <https://gallica.bnf.fr/ark:/12148/bpt6k2011211/f592.item.r=%22arithmo%20planimetre%22>

[NN1900-1] n. n.: **Dasnoy**, Arlon, Belgium, Annales de l'Institut archéologique du Luxembourg, no. 173, 1900, pages 173. Note: Dates of birth and death of Jean-Baptiste Dasnoy. Link: https://books.google.de/books?redir_esc=y&id=h4gBAAAAYAAJ&focus=searchwithinvolume&q=Dasnoy

[NN1914] n. n.: **Veteran Draftsman Is Discovered Dead in Bed**, The Washington Times, 22nd December, 1914, pages 5. Note: Obituary of Paul von Erichsen. See also [Eri1884]. Link: <https://www.loc.gov/resource/sn84026749/1914-12-22/ed-1/?sp=5&r=0.003,0.477,0.211,0.119,0>

[NN1983] n. n.: **Annales de Basses-Alpes**, no. 295-298, 1983, pages 106.

[Oza1684] **Ozanam, Jacques: La Géométrie Pratique Contenant La Trigonometrie Theorique & Pratique, la Longimetrie, la Planimetrie, & la Stereometrie**, Paris, 1684. Note: 323 pages. Link: <https://doi.org/10.3931/e-rara-1477>

[Sau1856] **Sauer, F.: Ein neues Planimeter von Buniakowski**, Buchdruckerei von J. Johnson, Mittheilungen der Kaiserlichen und freien ökonomischen Gesellschaft zu St. Petersburg, 1856. Link: <https://books.google.de/books?id=TrF5pAjDDDIC>

[Sch1891] **Schols, Ch. M.: Landmeten en Waterpassen**, Koninklijke Militaire Academie, Breda, 1891. Note: Plate XII, Figure 110 shows the Transformateur. Link: <https://books.google.de/books?id=0jtnAAAacAAJ>

[Sch1893-1] **Schlebach: Kloth's Flächenmaasstafeln**, Zeitschrift für Vermessungswesen, Vol. 22, no. 2 January, 1893, pages 60-61. Note: See also [Wil1892-2] and the response in [Klo1893]. Link: <https://archive.org/details/zeitschriftfrve23vermgooq/page/60/mode/2up>

[Sid1876] **Sidenblad, Elis: Swedish Catalogue O. Statistics, International Exhibition, 1876, Philadelphia**. 1903. Note: On page 98 J. P. Ljungström's cirkel planimeter is described. Link: <https://archive.org/details/swedishcatalogu00sidegoog/>

[Sta1835p] **Stabrowsky, Actuarius: Mathematisches Instrument zur Ermittlung des Flächen=Inhalts ebener Figuren**, Czerniejewo bei Gnesen, Polish patent number ?. Note: Cited in [NN1835-1]. Application filed 26.08.1835.

[Sta1858] **Stadler, Josef: Hyperbel=Planimeter**, Verlag von Friedrich Manz, Wien, Erfahrungen im berg= und hüttenmännischen Maschinen=, Bau= und Aufbereitungswesen, Zusammengestellt aus den amtlichen Berichten der k. k. österr. Berg=, Hütten= und Salinen=Beamten, 1858, pages 34-39, Tafeln XIV-XV. Link: <https://www.digitale-sammlungen.de/en/view/bsb10705800?page=40>, figures: <https://www.digitale-sammlungen.de/en/view/bsb10705819?page=56>

[Ste2009] **Stehouwer, Jan: De Transformateur**, De Hollandse Cirkel, Vol. 11, no. 3, 2009.

[Ten1828] **Tenner, Karl Christian: Kurze Beschreibung eines Planimeters oder allgemeinen Inhaltsmessers zum Gebrauch bei Landesvermessungen und für praktische Geometer, nebst einer Anweisung, wie auf eine leichte Weise eine sehr einfache Rechenmaschine verfertigt werden kann**, Leske, Darmstadt, 1828. Note: 24 pages with 3 copper plates. Cited in [NN1829-2].

[Tou2022] **Tournès, Dominique: Histoire du calcul graphique**, Cassini, Revue d'Histoire des Mathématiques, no. HAL Id: hal-03935854, 2022. Table of contents Link: <https://hal.science/hal-03935854>

[Usi1889-1] **Usill, George William: Practical Surveying**, Crosby Lockwood & Son, London, 1889, pages 280. Chapter "Computation Scale" pages 262-265. Further editions: 5 (1898); 14 (1932), 392 pages. Link: <https://books.google.de/books?id=S6aQUIGeCGgC>

[Vos2023] **Vossepoel, John: Mathematical Instruments – Sweden**, 2023. Link: <https://planimetria.jimdo.com/mathematical-instruments/sweden/>

[Wie1823] **Wießner, Gottfried: Beschreibung eines neuen mathematischen Instruments**, 1823. Link: <https://doi.org/10.3931/e-rara-14106>

[Wie1828] Wießner, Gottfried: **Beschreibung eines neuen mathematischen Rechnungsuhr zum Gebrauch bei den geometrischen Flächenberechnungen**, August Schmid, Jena, 1828. Bem.: Zitiert in [Zar1829].
Link: https://www.google.de/books/edition/Beschreibung_einer_mathematischen_Rechnu/-YVaAAAAcAAJ

[Wie1833] Wießner, Gottfried: **Beweis über Parallellinien, oder: dass alle drei Winkel eines jeden Dreiecks zusammengenommen zwei rechten gleich sind**, 1833.

[Wie1835] Wießner, Gottfried: **Praktische Anweisung zur Feldmeßkunst mit der Kette, dem Meßtische und Winkelspiegel zur Selbstbelehrung für alle, welche mit den ersten Lehren der Arithmetik und Geometrie nicht ganz unbekannt sind**, 1835. Note: 80 pages, 9 folded sheets.
Link: <https://doi.org/10.3931/e-rara-49147>

[Wik2023-1] Wikipedia: **Friedrich Körner (Mechaniker)**
Link: [https://de.wikipedia.org/wiki/Friedrich_Körner_\(Mechaniker\)](https://de.wikipedia.org/wiki/Friedrich_Körner_(Mechaniker))

[Wik2023-2] Wikipedia: **Jonas Patrik Ljungström**.
Link: https://sv.wikipedia.org/wiki/Jonas_Patrik_Ljungström

[Wik2023-3] Wikipedia: **Instrumentfabriks AB Lyth**.
Link: https://sv.wikipedia.org/wiki/Instrumentfabriks_AB_Lyth

[Wik2023-4] Wikipedia: **Tithe Act 1836**. Link: https://en.wikipedia.org/wiki/Tithe_Act_1836

[Wik2023-5] Wikipedia: **Léon Lalanne**. Link: https://en.wikipedia.org/wiki/Léon_Lalanne

[Wik2023-6] Wikipedia: **Viktor Bunyakovsky**. Link: https://en.wikipedia.org/wiki/Viktor_Bunyakovsky

[Wil1892-2] Wilski, P.: **Neue mechanische Rechenhülfsmittel**, Zeitschrift für Vermessungswesen, Vol. 21, no. 23, 1892, pages 625-630. Note: Describes Kloths hyperbolic plates. See also [Sch1893-1] and the response in [Klo1893]. Link: <https://archive.org/details/zeitschriftfrve14vermggoog/page/624/mode/2up>

[Wil2022] Williams, Alan: **Graphography: OK Computer**. (Updated 20th July 2022.).
Link: <https://graphography.org/index.php/2022/07/20/ok-computer/>

[Zar1829] Zaremba, Jan: **Planimetr narzedzie jeometryczne wymierzajace powierzchnia wszelkich figur prostokreslnych bez wykreslania i rachunku**, Typographie et Lithographie de J. Bourger, Arlon, 1829. Note: 46 pages. Link: <https://rcin.org.pl/dlibra/doccontent?id=7870>

[Zer2012] Zerfowski, Detlef, **Our World-wide Knowledge on Calculating Machines, Slide Rules and Calculating Tools. An Approach for a Comprehensive Overview of the Knowledge from Past until Present**, self-published, Bangalore, India, September 2012. Note: The book can directly be ordered from the author. With the book, free access to the continuously extended online database is granted (see <https://www.zerfowski.com/BibQuery/submitBibQuery.php?language=english>).

[Zer2021-4] Zerfowski, Detlef: **Early Area Measurement Instruments**, UKSRC Slide Rule Gazette, Vol. 21 June, 2021, pages 138-158. Note: Part 1 of this article.
Link: https://www.zerfowski.com/Papers/Zer2021-4_Early_Area_Measurement_Instruments.pdf